TIME-FREQUENCY LOCALIZATION OF ELECTROSTATIC DISCHARGE FOR LIGHTNING STUDY

S. Jitarwal[1]*, J. P. Pabari[1], D. Kumar[2], G. Dhote[3], S. Nambiar[1], Rashmi[1], T. Upadhyaya[4], K. Acharyya[1], V. Sheel[1] and Anil Bhardwaj[1],[1]Physical Research Laboratory, Ahmedabad, INDIA, (sonam@prl.res.in), [2]BITS Pilani, Hyderabad, [3]IEHE, Bhopal, [4]Charusat, Changa.

Introduction: Lightning is a large electrical discharge of very short duration of the order of few tens of microseconds that occurs in a planetary atmosphere. Though the lightning on Earth is well studied, it is not yet fully understood in the case of Venus. In case of Earth, water clouds are responsible for the lightning to occur; while in case of Venus, Sulphuric acid is an important constituent of the atmospheric cloud, at heights from ~47 to 65 km [1]. On Earth, lightning flash is mostly detected as cloud-to-ground discharge and ~20 % of the events are cloud-to-cloud discharge type [2]. However, the cloud-to-cloud lightning is more likely to occur on Venus [3]. To understand the lightning on Venus in detail, a Lightning Instrument for VEnus (LIVE) is proposed for future Venus mission [4]. The captured signal by the instrument is processed further to obtain more information of the detected lightning event. An efficient way of representation of the lightning signal is time-frequency localization [5, 6], which can be implemented using several transformation techniques like Short Time Fourier transformation (STFT), Hilbert-Huang transformation (HHT), Wigner-Ville distribution (WVD) and continuous Wavelet transformation (CWT) [7]. In the present work, various transformations are applied on a natural lightning pulse. We have captured this lightning pulse using LIVE at PRL during the past Monsoon season. We discuss which transformation provides better time-frequency localization for the captured lightning event based on their representation.

Time-Frequency Localization: A natural lightning discharge was captured earlier by the instrument, whose time domain signal is shown in Figure 1. Different transformations are applied on the time domain pulse to understand the representation.

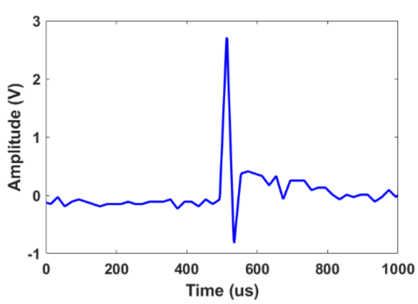

Fig 1: Natural lightning discharge pulse in time domain.

A) Short Time Fourier Transform: The Short Time Fourier transform is a fundamental tool for time-frequency analysis. It composed of two steps: first, the signal is divided into time segments and then, spectrum of each segment is obtained by Fourier Transform. This procedure called the STFT, leads to 3D representation which displays frequency content over time. Mathematically, the STFT can be expressed as [6]

$$S(t,f) = \int_{-\infty}^{+\infty} x(\tau)h(\tau - t)e^{-j2\pi f\tau}d\tau \qquad (1)$$

where, $h(t)$ is time window centered in t = 0 which is used to extract the time segments. The length of window $h(t)$ determines the time frequency representation of the signal.

(B) Wigner Ville Distribution: It provides excellent localization in time and frequency domains. The WVD focuses on decomposition of the signal energy in the time-frequency plane. It performs a correlation between the left and right folded part of the signal to find the overlap between the past and future values. If the time-domain signal is $x(t)$, then mathematically the WVD can be expressed as [6]

$$W(t,f) = \int_{-\infty}^{+\infty} x(t+\tfrac{\tau}{2})x^*(t-\tfrac{\tau}{2})e^{-j2\pi f\tau}d\tau \qquad (2)$$

However, the WVD is nonlinear and it is responsible for introduction of the interference terms. These interference terms can make the time-frequency representation challenging to interpret. In such case, Pseudo Wigner Ville Distribution (PWVD) is used, which includes a windowing function for removal of these interference terms.

(C) Hilbert Huang Transform: It is a nonlinear technique which extracts the time-frequency content of a non-stationary signal. This technique is comprised of two steps: (i) the decomposition of a signal into a sum of amplitude and frequency modulated wave using Empirical mode decomposition and (ii) the instantaneous amplitude *(a(t))* and frequency *(f(t))* are extracted using a demodulation technique. Finally, the time-frequency representation is obtained by displaying the time evolution of *a(t)* and *f(t)* of each the time-domain signal [4]. The analytical signal $Z(t)$ is defined as

$$Z(t) = x(t) + j\,H[x(t)] \qquad (3)$$

Where, $H[.]$ denotes the Hilbert Huang Transform. The instantaneous amplitude and frequency can be extracted from the analytical signal as follows

$$a(t) = |z(t)| \quad (4)$$

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}(\arg[z(t)]) \quad (5)$$

where, |.| and arg[.] operations denote the modulus and the argument of complex number respectively. The time-frequency representation is obtained by displaying the $a(t)$ and $f(t)$ of each intrinsic mode function in the time-frequency plane.

(D) Continuous Wavelet Transform: The CWT is another tool for time-frequency analysis. It is obtained by breaking up the signal into the shifted and scaled versions of a mother wavelet. Mathematically, the CWT is defined as [3]

$$C(t,a) = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(\tau).\psi(\frac{\tau - t}{a}).d\tau \quad (6)$$

where a is the scale and $\psi(t)$ is the mother wavelet. CWT provides the signal time evolution at different scales. The frequency of mother wavelet $\psi(t)$ having a central frequency f_c and scale a is given as $f = {f_c}/{a}$.

Simulation Results: In this section, we compare several time-frequency representations for a lightning discharge pulse, i.e., as depicted in Figure 1. The time domain signal has been analyzed using a Kaiser window and STFT, CWT, PWVD and HHT. All representations are given on the basis of the parameters like the window sample length (L), the shaping parameter (β), the overlap between adjoining sections (no) and the number of discrete points (n) with their values as 256, 5, 220 and 512, respectively. All transformations have been implemented using the MATLAB software, whose results are shown in Fig. 2 to Fig. 5, respectively. One can observe that by keeping all the parameters same for all the transformations, the STFT provides better resolution for the background with moderate resolution for the discharge event, the CWT and PWVD give poor resolution for the signal while the HHT provides best resolution for the discharge event with moderate resolution for the background.

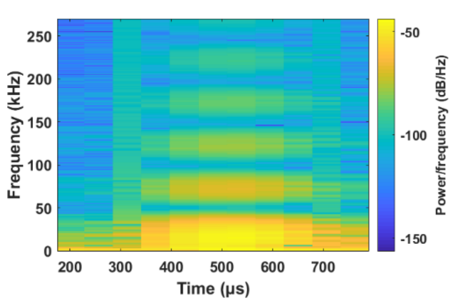

Fig 2: Time Frequency Representation of discharge pulse using STFT

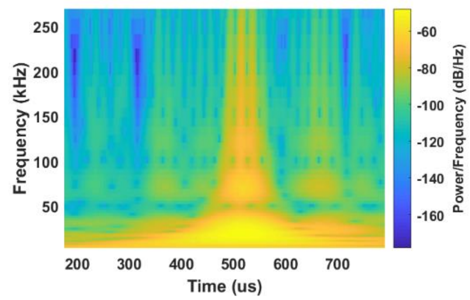

Fig 3: Time Frequency Representation of discharge pulse using CWT

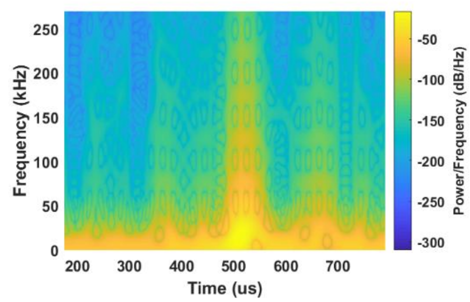

Fig 4: Time Frequency Representation of discharge pulse using PWVD

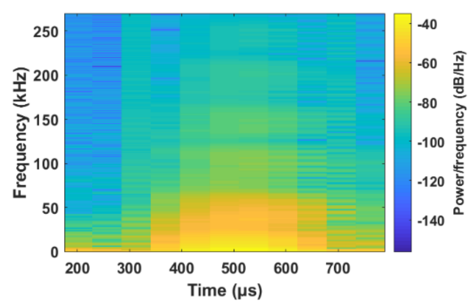

Fig 5: Time Frequency Representation of discharge pulse using HHT

Conclusion: This study has presented a comparison of several time-frequency localization techniques for a natural lightning signal, captured during the past monsoon season in Ahmedabad. Our results show that each technique is capable of providing time-frequency localization, however HHT gives easiness of interpretation or better resolution as compared to other transformations. Also, the realization complexity of the HHT is not very high. Thus, HHT provides higher resolution with lesser implementation complexity.

References: [1] Russell et al. (2011), *PSS*, 965-973. [2] Ullivi P. and Harland D. M (2014), *Springer*, 567. [3] Esposito L.W. et al. (1983) *Uni. Ariz. Press, Tucson*, 484-564. [4] Pabari J. et al. (2018) *LPSC XLIX*, Abstract # 1391. [5] Cohen L. et al. (1989) *IEEE*, 77, 941-981. [6] Flandrin P. et al. (1998), *Academic press.* [7] Bouchikhi E.H. et al. (2011), *IEEE*,3485-3489. [8] Huang N. et al. (1998), *Proc. Roy. Soc. London*, 454, 903-995.