THE FRACTAL NATURE OF PLANETARY LANDFORMS AND IMPLICATIONS TO GEOLOGIC MAPPING. S.J. Robbins\textsuperscript{a,1}. \textsuperscript{a}stuart@boulder.swri.edu, \textsuperscript{1}Southwest Research Institute, 1050 Walnut Street, Suite 300, Boulder, CO 80302.

Introduction: The primary product of planetary geologic and geomorphologic mapping is a group of lines and polygons that parameterize planetary surfaces and landforms. Many different research fields use those shapes to conduct their own analyses, and some of those analyses require measurement of the shape’s perimeter or line length, sometimes relative to a surface area. There is a general lack of discussion in planetary science of the fact that perimeters of many planetary landforms are not easily parameterized by a simple aggregation of lines or curves, they instead display complexity across a large range of scale lengths; in fewer words, many planetary landforms are fractals.

Because of their fractal nature, instead of morphometric properties converging on a single value, those properties will change based on the scale used to measure them. Therefore, derived properties can change – in some cases, by an order of magnitude or more – just when the measuring length scale is altered. This can result in significantly different interpretations of the features. In this abstract, and at the Planetary Mappers’ Meeting, I will discuss applications of fractals to problems of planetary mapping, interpretation, and related scientific investigations.

Application to Impact Crater Ejecta: The extent of crater ejecta can be an important component of mapping, but researchers who study crater ejecta typically want to characterize it further. Morphometric properties are almost exclusively derived from tracing the perimeter and deriving different metrics from that perimeter trace. These properties include: Perimeter, area, lobateness ($\Gamma$; perimeter divided by the circumference of a circle with the same area), extent of the continuous ejecta from the rim, and ratio of that extent to the crater’s size. Except area, all these metrics are affected by the fractal nature of the ejecta.

Figure 1a shows a simulated example of a complex pattern that has some similar properties to a real ejecta deposit. Figure 1b shows the measured perimeter and area of the shape (vertical axes) as different measuring length scales were used ($\epsilon$, horizontal axis). Area converges to a single value, but there is no steady-state perimeter; in fact, the perimeter increases significantly at the smallest measuring lengths due to the extra complexity at those small scales. Therefore, $\Gamma$ (Fig. 1c) also changed depending on $\epsilon$, and that change was non-linear, not predictable, nor easily parameterized.

An additional problem of changing $\epsilon$ is two features that are identical in every way except scale, such as hypothetical ejecta from a 1 km diameter crater versus 10 km diameter crater, will have different perimeter versus area properties when measured with the same length scale. This is because more intricate features resolved at 10 m for the larger crater will not resolve at 10 m for the smaller crater. Figure 1b illustrates this result with the example feature scaled up by a factor of 10, but keeping $\epsilon$ the same (the perimeter results are scaled to have identical values for $\epsilon \approx 0.3$, and the areas are scaled to have the same asymptotic values). The general trend is the surface area converges more quickly, but the perimeter again has non-linear behavior. (Non-linear behavior occurs at different $\epsilon$ because the shape is not a true fractal.) Therefore, $\Gamma$ (Fig. 1c) is different for a given $\epsilon$ despite the identical nature of the underlying shape.

Application to One-Dimensional Features Studied in Two Dimensions: Valley networks and properties associated with them also should be considered in the context of fractals, such as stream order (branching complexity, known in mathematics as the Strahler number or Horton-Strahler number), stream length, and drainage density (valleys per unit area). These three properties have important implications for the valley network and the environment in which it formed. From Viking-era imagery, [1] produced maps of the valley network distribution on Mars. From a new generation of spacecraft imagery, [2] identified many more, smaller valleys that were not visible in previous imagery, resulting in $8 \times$ more valleys, $2 \times$ more length, $2 \times$ higher stream order, and in some cases over $10 \times$ higher drainage density. The limiting factor here was primarily image resolution and quality, but the difference in detail between the valley network maps is similar to detail one observes when examining fractals at finer scales. This emphasizes the need to remember “scale-based geologic mapping” is scale-based for a reason, and all interpretations and conclusions must be couched in caution due to this limiting scale and potentially a lack of higher quality data.

Application to Two-Dimensional Features Studied in Two Dimensions: Ballistic and flow processes such as lava emplacement – often display fractal-like geometries. The fractal nature of lava flows were studied extensively in the 1990s [3], and they demonstrated that plotting the measured perimeter versus measuring scale length yielded different and potentially diagnostic power-law fits, thus giving a diagnostic tool for differentiating between lava flow types simply from analyzing the fractal nature of the mapped units – and therefore a tool for understanding the underlying material properties and geophysics.

Issue of Data Scale and Resolution: A problem with remote sensing data is data fidelity: Whatever data are being analyzed exist at a finite resolution, and in practice this will limit the extent of the analyzable scales. Fractal characterization requires analysis at a range of scales, but that may not be possible given the data scale or resolution. There are significant practical
issues here, where it is possible that one may not be able to recognize whether a feature displays fractal-like characteristics. In such cases, it is the recommendation that researchers acknowledge this potential deficit. This was described above with respect to valley networks, but it also applies to such investigations as slopes (limiting fidelity of an elevation model can soften slopes), thermal inertia (a pixel is the average of all thermal inertia properties of exposed material in that pixel), and other studies.

**Suggestion—Compute Fractal Dimension:**
Fractal dimension provide a simple metric to characterize the inherent complexity of a shape, and there are many methods to calculate $D$. A simple linear example starts by using $\epsilon = 1$. One might measure a river as having a length of $N = 3$ (length = “$N$” in fractal studies). If the length scale is decreased by $3 \times (\epsilon = 1/3)$, one would expect the same $N = 3$, but instead $N = 4$ due to the meandering nature of the river. The fractal dimension can be calculated in two ways. The first method uses only the data at that point: $D = -\log(N) / \log(\epsilon) \approx 1.262$. Alternatively, one can graph $\log(N)$ versus $\log(\epsilon)$, and the fractal dimension is $1 - \text{slope}$, where “slope” is the slope of a best-fit line. In this example, the plotted values would be $x = \{\log(1/3), \log(1\ldots)\} \approx \{-0.477, 0\}$ and $y = \{\log(4), \log(3)\} \approx \{0.602, 0.477\}$, such that the best-fit line is $y \approx 0.477 - 0.262 \cdot x$, and the fractal dimension $D \approx 1 - (-0.262) \approx 1.262$, in agreement with the other method.

**Conclusion:** A final consideration is a practical one: Despite the examples described, the most common component of mapping in the planetary science community is the basic identification and tracing of physiographic units at a certain, singular scale, and how fractal considerations apply or could be applied may not be clear. In practice, one can take a mapped unit and compute the fractal properties. For purpose, it is my opinion that the fractal nature of units will rarely affect the actual process of mapping, for most mappers are interested in the area covered by units, and areas converge quickly to a single result. However, mappers often care about the perimeter of units, and in that measurement fractal considerations are important. Additionally, mappers usually want other researchers to use their product, and those other researchers may need to conduct analyses where the fractal nature of landforms will affect results. Additionally, as referenced with types of lava, directly characterizing the fractal nature of the perimeter can lead to important conclusions in themselves, which can affect the interpretation of mapped units.

The community should consider revisiting the fractal implications to mapping and especially to studies that rely on morphometric properties of those mapped units.

**References:**

**Funding:** This work was funded in part by MDAP award NNX15AM48G.

**Publication:** This work has been accepted for publication: Robbins, S.J. (in press) “The Fractal Nature of Planetary Landforms and Implications to Geologic Mapping.” *Earth & Space Sci.*

**Figure 1:** Simulated complex shape (☉), the unitless measured perimeter and measured area (☉), and derived lobateness (☉) when shape is scaled from a large (open circles) to small (filled circles) size. The shape was created once, and the measuring length scale was adjusted in software (one can derive $D$ done with the original perimeter by using software).