Initial velocity distribution and consequent spatial distribution of fragments

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ABSTRACT

A fragmentation on orbit is a complex event, yielding numerous pieces in a range of sizes, masses, energies and subsequent orbits. Since exact knowledge of the fragmentation is unobtainable, it is common to use statistical methods to describe these attributes as they apply to the set of fragments. There are numerous hypotheses and arguments that have been made from which the statistical distributions of these attributes can be inferred, and in some cases, can be supported or weakened by observations, but given the complexity of the fragmentation, the subsequent dynamics, and the difficulty of making thorough observations, there is not much definitive that can be said.

Recently, the authors have developed a method for propagating orbital densities, as opposed to points, that produces exact spatial distributions from a distribution in velocity at a single location on orbit (Healy, Liam M., Christopher R. Binz, and Scott Kindl. “Orbital Dynamic Admittance and Earth Shadow.” The Journal of the Astronautical Sciences, January 3, 2019. https://doi.org/10.1007/s40295-018-00144-1). In that work, large-scale geometric traits of debris evolution, such as the pinch point, anti-pinch line, and radial bands, are shown to be present regardless of the initial velocity distribution, and so must be due to orbital dynamics. In this and related publications, the authors have used as initial velocity distribution various constant density distributions, with and without maximum velocities, and the NASA 2001 orbital debris distribution, which, being exponential, has no maximum velocity. The most prominent observable feature from the evolved spatial distribution attributable to the initial velocity distribution is the maximum velocity.

Given the complexity of the evolved distribution and the apparent lack of information on the initial distribution that persists in the observable result, the various models, hypothesis, and assumptions taken must be carefully examined. Conversely, if the primary goal of making these models, hypotheses, and assumptions is to deduce the consequent spatial distribution, then in large part it does not matter what they are or even if they are self-consistent.

It is most convenient to divide the relative velocity at fragmentation into two parts based on a spherical polar decomposition: the direction and magnitude (i.e., speed). A completely isotropic distribution, commonly assumed, has no preferred direction, and is completely spherically symmetric. At the opposite extreme, a completely anisotropic distribution has all delta-v pointed in a narrow range of directions. There are numerous applications that need so-called *directional* statistics, that is statistics on spheres of various dimensions, and this has blossomed into small but significant subfield of statistics. One can approximate the anisotropy with the available models. In this work, we examine the effect on the spatial distribution and the degree to which the velocity anisotropy may be deduced from this distribution.

The speed distribution may be asserted from various analogues in gas dynamics, e.g. Maxwell-Boltzmann distribution, or from a Gaussian distribution of velocities. The latter than leads to the Rayleigh distribution in speed, and this is different from the Maxwell-Boltzmann distribution. Here, we explore the difference (subtle, even when looking directly at speeds) in the spatial distribution.

A speed distribution can be derived from a mass distribution and an energy distribution. It is widely accepted that mass distributions follow a power law, but the exponent of that power law may be open to debate. Energy distributions are assumed to follow the equipartition principle, namely, that each fragment gets the same amount of energy, but it should be noted that not all the available energy goes into kinetic energy. In this work, we examine the relationship of these distributions, in particular the exponent in the mass power law, with the speed distribution and discuss its potential observability in the spatial distribution of the fragments.