Studying a direct re-entry from a Sun-Earth Libration Point Orbit: can ground uncertainty be kept under control?

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ABSTRACT

In recent years, there has been an increasing attention to Libration Points Orbits (LPOs) in the Sun-Earth and Earth-Moon systems, because they represent a fundamental resource to obtain astrophysical and planetary observations. Even for this kind of missions an awareness of the end-of-life opportunities is crucial. In the present work, we keep on investigating the opportunity of an Earth’s atmospheric re-entry.

In the past, we have investigated this possibility in terms of trajectory design and ground casualty risk for three specific missions (SOHO, Herschel and Gaia). We showed that low-cost solutions can exist in terms of Δv-budget, according to the size of LPO, that the steep re-entry and the shorter period in the atmosphere cause the debris field to be significantly shorter in length and closer to the point of breakup than those associated with circular re-entries, and that, as in the case of circular re-entries, it is not possible to provide globally applicable recommendations in terms of demise.

In this work, we aim at addressing the aspect related to the orbit determination and maneuver uncertainty to constrain the impact location. The sensitivity study performed in the past showed that the longitude of impact is linearly dependent to the time of flight, while the latitude depends on the inclination corresponding to the LPO and the osculating argument of pericenter at the entry into the atmosphere.

After a review on the tracking procedures, we show how the Semi-Linear Method, usually applied in the context of asteroid dynamics and impact monitoring, could be used to study the ground uncertainty in the re-entry of LPO spacecraft. Moreover, we will simulate observations corresponding to a direct LPO re-entry in order to understand how the orbit determination campaign shall be conducted to bound the ground uncertainty.

1 INTRODUCTION

Nowadays the mission design has to take into account the implementation of end-of-life disposal solutions to preserve the space environment and for the sustainability of the project as a whole. The Libration Point Orbit (LPO) missions do not escape this rule. Such missions are defined as those in which the spacecraft revolves around one of the five Lagrangian points of the Sun-Earth system. These points of equilibrium in the RTBP (Restricted Three Body Problem) have the property of being at rest with respect to a pair of primaries, which makes them particularly useful for a number of space applications. More specifically, LPOs around the collinear libration points L1 and L2 ([1]) have very well-known advantages in terms of thermal stability, observations and communication geometries, and a minimum level of required budget for on-orbit maintenance maneuvers ([2]). For these reasons, the number of LPO missions around these two points has increased over the last 15 years and a further growth is expected within the next years. The dynamical instability around L1 and L2 produces a behavior for which an uncontrolled spacecraft at the end of its life will leave its operational orbit. This fact suggests that no protected regions have to be defined for LPO missions. Nonetheless, long-term propagations have highlighted the possibility for these departing trajectories to come back to the Earth–Moon system, with potential interferences with LEO and GEO protected regions, as well as re-entries in the Earth’s atmosphere. In [3] and [4], three different options for the disposal of LPO missions are considered: Earth re-entry, lunar impact, disposal on graveyard orbits. In [5] and [6], the first possibility, in terms of trajectory design and ground casualty risk for three specific missions (SOHO, Herschel and Gaia), was investigated. It was shown that low-cost solutions can exist in terms of Δv-budget, according to the size of LPO, that the steep re-entry and the shorter period in the atmosphere cause the debris field to be significantly
shorter in length and closer to the point of breakup than those associated with circular re-entries, and that, as in the case of circular re-entries, it is not possible to provide globally applicable recommendations in terms of demise. In particular, in [6] the case of SOHO has been studied, in terms of orbit uncertainties and risk on the ground, confirming that the longitude of impact is linearly dependent to the time of flight, while the latitude depends on the inclination corresponding to the LPO and the osculating argument of pericenter at the entry into the atmosphere.

In this paper, after a review on the tracking procedures and re-entry observations (Section 2), we will focus on the uncertainty on the ground for a LPO mission re-entry. Such kind of re-entry dynamics (Section 3.1) is quite similar (due to the high velocity) to the one of a minor natural body on a collision course with the Earth. Due to this similarity, we will apply to this class of problems the tools used for the Impact Monitoring of Near-Earth Objects (NEOs, [7]), to map the orbital uncertainty on the ground by means of the Semi-Linear Method (SLM, Section 3.2). Our case study will be SOHO, whose nominal orbit is an L1 halo orbit with an in-plane amplitude of about 340,000 km and an out-of-plane one of about 170,000 km. In Section 4, we will show the results of our simulations, that consider the possibility of an uncontrolled re-entry, that is the worst possible scenario. In Section 5, we will draw some conclusions and we will highlight potential future work to do.

The simulations have been performed used a modified version of the OrbFit software (adams.dm.unipi.it/orbfit), starting from re-entry trajectories computed in [5]. OrbFit is a free (distributed under GPL license) Orbit Determination (OD) software system developed to compute the orbits of asteroids starting from the observations. It has the capability of propagating orbits and computing predictions on the future (and past) position on the celestial sphere. OrbFit can be used to find a known asteroid, to recover a lost one, to attribute a small group of observations, to identify two orbits as the same, to study the future (and/or past) close approaches to Earth, then to assess the risk of an impact. There exists also modified versions of OrbFit (not distributed) to handle geocentric objects and imminent impactors ([7]).

2 TRACKING PROCEDURES

2.1 Routine tracking and OD uncertainty

In general, LPO missions, during the activity phase, are tracked by Earth radio stations. Concerning our test case, SOHO is operated from NASA’s Goddard Space Flight Center (GSFC) near Washington. There is an integrated team of scientists and engineers from NASA, partner industries, research laboratories and universities working under the overall responsibility of ESA. Ground control is provided via NASA’s Deep Space Network (DSN) antennae, located at Goldstone in California, Canberra in Australia and Madrid in Spain (Credits: https://www.esa.int/Science_Exploration/Space_Science/SOHO_factsheet).

We can consider, as done in [6], that the orbit of the spacecraft is quite well determined by the routine tracking, assuming an uncertainty in position up to 70 m, and one in velocity up to 0.4 mm/s.

2.2 Re-entry tracking and observations

During the re-entry phase, we can imagine two different scenarios: the radio link is still active or the re-entry is uncontrolled and only optical and radar observations are available. In the following, we will consider the second and worst scenario, trying to understand if the ground re-entry uncertainty can be kept under control.

Concerning the possibility to observe the spacecraft during the re-entry phase, in [8] the authors outline a strategy to achieve complete observational coverage of an incoming object on short notice. They suggest that it is necessary to develop a network of instruments of different classes, each of which is capable of providing a given type of observation coverage. For this need, it is necessary to have access to one large telescope, one or two mid-class telescopes (used with much more flexibility and for multiple nights, but less sensitive at large distances) and a network of small telescopes for covering the last phases of the impact trajectory from multiple locations. Thus, the observational campaign can be divided in two phases: a long arc coverage at large geocentric distances, carried out with middle to large class professional telescopes; a short leg refining phase to be executed in the days before impact, where astrometric quality and number of observing stations are essential to constrain the impact trajectory.
3 RE-ENTRY DYNAMICS

3.1 Dynamical models

As described in [6], the transfer is established on the unstable invariant manifold of the nominal LPO, starting from a first approximation computed in the Circular Restricted Three–Body Problem ([1]) applied to the Sun–Earth system. In the full model, the trajectory persists, and the relative long time required to leave the LPO due to the asymptotic nature of the hyperbolic manifold is overcome by suitable maneuver. The transfer can be achieved either directly, and in this case it can last about 3 months, or after some revolutions about the Earth. In the latter case, the perigee altitude is generally always outside the Low Earth Orbit (LEO) region, and the spacecraft can experience a lunar close approach, or an excursion through the L1/L2 libration points neighborhood. In all the cases, the geocentric osculating orbit at the final re-entry tends to a parabolic orbit, having an eccentricity of about 0.99 and a semi-major axis of the order of 600,000 km. The re-entry speed in about 11 km/s, thus the time spent in the LEO region is very short and the atmospheric uncertainties play a minor role in the dynamics ([5]). Due to these features, the idea is to consider a hypervelocity re-entry from LPO as a natural object on a collision course with the Earth, trying to use the algorithms developed for Impact Monitoring of NEOs ([7]).

3.2 Semi-Linear Method (SLM)

The Semi-Linear Method (SLM) was born in the field of dynamical astronomy for the sky recovery of lost asteroids. The idea was to map the orbital uncertainty to the celestial sphere of a future epoch ([9]). But, as explained in [10], such kind of mapping can be used also for impact risk assessment problem, projecting the uncertainty on the Target Plane (TP), that is a plane containing the center of the Earth and orthogonal to the velocity of the object. More recently ([10]), these algorithms have been used to compute the boundary of the impact region of an asteroid on ground, defining the so-called impact corridor, at different levels of uncertainty.

Our idea is to apply these concepts to the dynamics of a man-made object re-entry and to compute the impact corridor in this case. The starting point is a full least squares solution \((x_0, \Gamma_s)\) having probability 1 to experience an Earth re-entry. The SLM consists essentially in following three computational steps, using the impact map \(F^b\) (for more mathematical details, see [11]). This function is defined in a neighbourhood \(W\) of \(x_0\) and it maps the orbit on the impact surface \(S_h\), that is the set of points in space at altitude \(h\) above the WGS 84 Earth reference ellipsoid. The three steps (a graphical sketch is represented in Figure 1) are the following.

I. The confidence region \(Z_{in}^X(\sigma)\) is linearly propagated using the Jacobian \(DF^b\) of \(F^b\) at \(x_0\): the result is the linear confidence region \(Z_{in}^Y(\sigma)\) on the tangent space to \(S_h\) in \(F^b(x_0)\).

II. The linear approximation given by \(DF^b(x_0)\) is exploited to select a representative curve \(\xi_X(\sigma)\) on the boundary of the initial confidence ellipsoid \(Z_{in}^X(\sigma)\), that is an ellipse.

III. A finite sampling of the ellipse \(\xi_X(\sigma)\) is propagated using a non-linear model (including all the relevant perturbations), in order to obtain the predicted semi-linear boundary at altitude \(h\).

![Figure 1: graphical sketch of the semilinear method, courtesy from [11]](image.png)
4 SIMULATIONS

The test case for our simulations is SOHO (Solar & Heliospheric Observatory, ESA-NASA joint mission), like in our previous paper [6], because it is still operational, and because the designed re-entry strategy requires only one maneuver. SOHO was launched in 1995, it is currently orbiting around the Sun on a nominal L1 Halo orbit with an out-of-plane amplitude of about 120,000 km. Figure 2 shows three re-entry transfers, one direct and two non-direct. The solution in purple is direct, that is the spacecraft arrives to the Earth before any other close approach. The solution in green approaches the Earth-Moon neighborhood three times before re-entering. The solution in light blue first gets to an altitude of about 120,000 km, then it gets away to return down to about 210,000 km, and ultimately reaches the Earth at a high latitude after a journey in the L2 region.

Figure 2: Re-entry trajectories in the Sun-Earth synodical reference system. Three different types of re-entry are considered. The solution in purple is direct, the one in green is non-direct and one in light blue is non-direct at high altitude. Courtesy from [6].

The dynamical analysis performed in [6] suggests that, at the very beginning of the mission planning, it should be taken into account also the end-of-life procedure, in order to meet a reasonable tradeoff among mission objectives, operational constraints and end-of-life opportunities. In the case of LPO, from a dynamical point of view the crucial parameters are the size and type of the nominal orbit, and the Δv on-board. The size and the type of LPO determine if an almost natural re-entry can be achieved, and the maximum inclination achievable by the re-entry trajectory. This study has suggested that the role of the argument of pericenter and its evolution along the transfer has also to be carefully investigated in the future to control the re-entry latitude. On the other hand, the Δv on-board is used to subdue the asymptotic dynamics of the orbits, to select a direct or non-direct re-entry and, when needed, to design a controlled re-entry. For SOHO, the authors in [6] recommend a Δv-budget of order of tens of m/s at the end-of-life. This value is suggested by the specific departure LPO studied, assuming that the re-entry latitude is allowed to be relatively low. Moreover, in [6] it was highlighted that the uncertainty in the maneuver required to target the Earth, even if small, can alter dramatically the re-entry conditions, in particular the probability of success and the longitude of arrival. To overcome the high spread in the longitude of impact and control the re-entry location, a second maneuver can be required. However, in the following, we will consider just one maneuver and in a future work we will investigate the possible reduction of ground uncertainty by applying a second maneuver. Moreover, for our purpose, we will consider a direct re-entry.
The idea is to perform three classes of simulations and plotting the results using Google Earth. The three classes are the following:

- assuming only the uncertainty due to the OD (Sec. 4.1);
- adding the uncertainty due to the maneuver (with different levels if uncertainty, Sec. 4.2);
- adding simulated ground optical observations in the worst scenario (Se. 4.3).

In all the cases, we start from a set of Cartesian coordinates corresponding to a re-entry trajectory (computed using the methods described in [5]) and we propagated it until the epoch of re-entry using the OrbFit propagator.

### 4.1 Only OD uncertainty

In this first simulation we assume a direct re-entry orbit with its uncertainty due to the OD (see Sec. 2.1). This case has essentially the goal of proving the consistency of the algorithm that uses the SLM.

- Epoch initial conditions: MJD 59074 August, 13th 2020
- Re-entry: November, 14th 2020
- Time of flight: 93 days

![Impact corridor, located in the Atlantic Ocean. Top: h=100 km and h=0 km (up to 5-σ). Bottom: zoom on h=0 km (up to 5-σ).](image)

The red zone is the uncertainty region with 1-sigma uncertainty.

Figure 3 shows the impact corridor on the surface of the Earth. The re-entry is located in the Atlantic Ocean. On top, two different regions of uncertainty are shown: the one in green is the impact corridor (up to 5-σ) for an altitude of h=100 km, while the one in red/yellow is the impact corridor (up to 5-σ) for an altitude of h=0 km. On bottom, there is a zoom on the impact corridor on the ground (the red zone is up to 1-σ). We can note that the uncertainty is essentially spread in longitude and is quite small (1’ at 1-σ level), in agreement with the results obtained in [6] using a Monte Carlo technique.

### 4.2 OD and maneuver uncertainties

Now we consider the same re-entry trajectory, with the usual OD uncertainty, but we add some uncertainty due to the maneuver. The epoch of re-entry does not change significantly, as the time of flight.

- Initial conditions: MJD 59074 August, 13th 2020
- Re-entry: November, 14th 2020
• Time of flight: 93 days

We present three different levels of $\Delta v$ uncertainty, the third one corresponding to the 1% of the nominal re-entry maneuver.

**I level: $\Delta v$ uncertainty about 7.52 mm/s**

The impact corridors, for the two altitudes $h=100$ km and $h=0$, and up to 5-$\sigma$ uncertainty, are shown in Figure 4. As in the previous section, the uncertainty is essentially spread in longitude and it is about 4 degrees at 1-$\sigma$ level, compatible with the Monte Carlo results described in [6].

![Figure 4: Impact corridor, $\Delta v$ uncertainty about 7.52 mm/s, nominal impact location in the Atlantic Ocean. Green: $h=100$ km (up to 5-$\sigma$) Red: $h=0$ km (up to 5-$\sigma$)](image)

**II level: $\Delta v$ uncertainty about 7.52 cm/s**

The impact corridor is represented in Figure 5. The uncertainty regions for the two altitudes $h=100$ km (green) and $h=0$ (red), 1-$\sigma$ level, are shown. Even at this level, the uncertainty is essentially spread in longitude and it is about 32 degrees at 1-$\sigma$ level. As expected, the uncertainty on ground increases, although the nominal impact location is nearly the same of the previous cases.
Figure 5: Impact corridor, $\Delta v$ uncertainty about 7.52 cm/s, nominal impact location in the Atlantic Ocean.
Green: $h=100$ km (up to 1-$\sigma$). Red: $h=0$ km (up to 1-$\sigma$).

**III level: $\Delta v$ uncertainty about 75.2 cm/s**

As pointed out by Figure 6, such level of uncertainty creates a quite critical situation. The uncertainty in longitude at 1-$\sigma$ level is almost as long as the entire circumference. Now, the crucial issue is to understand if an observational campaign could contribute to reduce the ground re-entry uncertainty. As highlighted in Sec. 2.2, there are different ways to perform an observational campaign during the re-entry phase. In the following, we will explore only one of these possibilities, that is a long arc coverage at large geocentric distances, carried out with a large professional telescope.

Figure 6: Impact corridor, $\Delta v$ uncertainty about 75.2 cm/s. The uncertainty region wraps on almost all the Earth circle.
Red: $h=0$ km (up to 1-$\sigma$).
4.3 Adding observations

The scenario is the one corresponding to the third level of uncertainty studied in the previous subsection. In order to study the possibility to reduce the uncertainty on ground, we followed these steps (the initial input is the re-entry trajectory with the covariance containing the maneuver uncertainty of about 75.2 cm/s).

- We simulated 15 optical observations from the Catalina Sky Survey (a professional telescope) spanning an arc from August 14th (the day after the re-entry maneuver) to September 30th.
- We computed a new orbital solution with its own covariance matrix using a differential correction procedure.
- We propagated this new solution and we checked that it was a re-entry trajectory still. The day of re-entry was the same.
- We computed the impact corridor on ground using the SLM.

Figure 7 shows the result, at 1-σ level, of the previous algorithm. The uncertainty results smaller than the previous case, 64 degrees in longitude.

![Figure 7: Impact corridor, Δv uncertainty about 75.2 cm/s, but some additional ground optical observations.](image)

Red: h=0 km (up to 1-σ). Green: h=100 km (up to 1-σ).

This simulation proves that, even in catastrophic cases due to a big uncertainty at the time of re-entry maneuver, an observational campaign could be used to bound the impact location.

5 CONCLUSIONS AND FUTURE WORK

Re-entry trajectories to the Earth have been recently considered as a valuable end-of-life option also for Sun–Earth LPO missions. For this reason, some previous studies investigated high velocity highly elliptic re-entries from Lagrangian points, in terms of orbit uncertainties and risk on the ground, using essentially a Monte Carlo analysis.

In this paper, assuming that the dynamics of such objects is quite similar to the one of a natural body on a collision course with the Earth, we try to apply the algorithms used in the field of impact monitoring to a problem of different nature. In particular, we proved that the Semi-Linear Method is a valuable alternative algorithm for handling the ground re-entry non-linearity. Moreover, we showed that, in case of non-small uncertainty due to the re-entry maneuver, it is possible to better constrain the impact location using optical observations by Earth telescopes. Obviously, this is a preliminary work, but promising. There will be a lot of work to do in future, including an ad-hoc development of the OrbFit software.
Concerning new tests to perform we would like:

- to investigate the dependence of the ground uncertainty on the time of observations;
- to add simulated radar observation to understand if the uncertainty can be drastically reduced;
- to study how a maneuver during the re-entry phase could modify the uncertainty and the impact location.

6 REFERENCES