Atmospheric density gradient torque - a possible new torque estimated from the rotational state of Tiangong-1

Hou-Yuan Lin\(^{(1,2)}\), and Chang-Yin Zhao\(^{(1,2)}\)

\(^{(1)}\) Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210034, China  
\(^{(2)}\) Key Laboratory of Space Object and Debris Observation, Chinese Academy of Sciences, Nanjing 210034, China

ABSTRACT

Tiangong-1 ended its service in March 2016 and re-entered the atmosphere in April 2018. We organized a joint observation from November 2017 to April 2018 to estimate its rotational state. The angular momentum of Tiangong-1 was relatively stable during the observation period, but its rotation rate was found to increase, which was an unexpected phenomenon. Because the rotational acceleration significantly increased with decreasing orbital altitude, which is consistent with the increase in atmospheric density with decreasing orbital altitude, we propose a new torque called atmospheric density gradient torque that considers the torque generated by the change in atmospheric density with the orbital altitude at the satellite scale. The order of magnitude of this new torque is estimated in line with the increasing rate of angular momentum. The atmospheric density gradient at the position of Tiangong-1 after February 2018 increases faster, which is also consistent with the increasing accelerated rate of the rotational speed. The numerical results indicate that the new torque model provides a nonnegligible effect but cannot fully describe this acceleration. The atmospheric density gradient torque model, as well as the aerodynamic model, may need improvement by addressing minor factors omitted in previous models.

1 INTRODUCTION

The first prototype Chinese space station, Tiangong-1, re-entered the atmosphere on April 2, 2018, at UTC 0:15. Before its reentry, we organized an optical and laser joint observation of Tiangong-1 from November 2017 to April 2018. Derived from the variation in the relative distance between two laser reflectors on Tiangong-1, the evolution of the rotational state was well estimated [1]. These data are currently unique data for targets below 300 km and have been published in the Zenodo database [2]. From the data, we found an unexpected rotational acceleration of Tiangong-1, and the acceleration effect is significant. We deeply suspect that this acceleration torque is related to the atmosphere. Hence, we proposed an atmospheric density gradient torque (ADGT) model [1]. This paper will elaborate on the details of the modeling and numerical simulation of ADGT.

2 MODEL OF ATMOSPHERIC DENSITY GRADIENT TORQUE

\[ \text{Fig. 1. [1] Atmospheric density gradient torque. The shadow shows that the atmospheric density } \rho \text{ decreases with increasing orbital altitude. The atmospheric force } df_1 \text{ at the end of the satellite nearest the Earth is greater than } df_2 \text{ at the far end, which generates the torque to accelerate spinning.} \]
A sketch of the ADGT is shown in Fig. 1. The concept of this torque is derived from the gravity-gradient torque. It is well known that the atmospheric density decreases with the orbital altitude, and this process has a certain gradient, even at the satellite scale; specifically, the atmospheric density at the far end will be slightly higher than at the near end of a satellite. When the satellite moves forward, the atmospheric forces acting at both ends are different, thus generating a torque. For Tiangong-1, the sense of ADGT is the same as that of rotation, and then the rotation is accelerated.

Of all the common torques, conservative dissipating torques obviously do not cause such a change. The torques similar to the light pressure torque, which are independent of position and angular velocity, can only linearly increase the rotational speed. The evolution of the rotational speed of Tiangong-1 indicates that the rate of increase in rotation was greater than an exponential increase. In the corresponding time, the parameters that best reflect this change were the orbital altitude, atmospheric density and its gradient with altitude. A strong correlation can be found in Fig. 2 between the rotational speed and the atmospheric density gradient, where the gradient is obtained by using the MSIS90 model to calculate the density difference between the values at the satellite location with an altitude difference of 10 m. Therefore, the ADGT was proposed.

Such a model may seem ridiculous because the difference in atmospheric density over a few meters is very small, but the torque produced is significant. We need to estimate the order of magnitude of this new torque.

In Fig. 2, from Dec. 1, 2017, to Feb. 1, 2018, the increase in angular speed was

\[ \Delta \omega \sim 0.2 \, ^\circ/\text{s} \]

\[ \Delta t \sim 60 \, \text{Day}. \]

Then the mean variation rate of the angular momentum was

\[ \frac{\Delta H}{\Delta t} \sim 5 \times 10^{-6} \, \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}. \]

In this case, the classical atmospheric drag formula can be applied as follows:

\[ \Delta F = -\frac{1}{2} C_d A v^2 L d \rho \]

where the velocity is slightly larger than the first cosmic velocity, the length is half of the size of the satellite and solar panels, the area is half of the mid-value of the largest section (30 m²) and the smallest section (10 m²), and the atmospheric density gradient is assigned the value obtained on Jan. 1st, 2018. Then, the ADGT is
which is the same order of magnitude as the rate of increase of the angular momentum. This finding suggests that our torque model is reasonable.

3 NUMERICAL SIMULATION

Numerical simulation is performed to validate the proposed torque model. In the simulation, the Tiangong-1 model is discretized into surface elements, and the atmospheric force acting on each element is calculated (Sect. 3.1). Actual orbital and space environment parameters are used in the numerical simulation. To improve the computational efficiency, the orbital elements are reduced (Sect. 3.2) and not numerically integrated. We tested the precision of the orbital approximation, the refinement of the Tiangong-1 model, and the effectiveness of the integration interval to ensure that these parameter values did not produce numerical errors that would affect the results.

3.1 Simulation of the action of the atmosphere

We reduced Tiangong-1 into a cylinder with a length of 10 meters and a radius of 1.6 meters and two 7 m × 3 m plates. An element is established every 0.5 meters, as shown in Fig. 3, where the circular portion of the cylinder is divided into 20 points to ensure that the spacing is roughly equal. We use molecule-surface interaction model [3] as the aerodynamic force acting on each unit surface element, which is expressed as

\[
d\mathbf{f} = -\rho v^2 \cos \theta_i ((2 - \sigma_N) \cos \theta_i \hat{n} - \sigma_T \sin \theta_i \hat{\tau}) dA,
\]

where \( \hat{n} \) is the normal of the surface, \( \hat{\tau} \) is the tangential vector, \( \theta_i \) is the incident angle, \( dA \) is the area of a unit element, and \( \sigma_N \) and \( \sigma_T \) are the normal and tangential momentum exchange coefficients, respectively, which both take an empirical value of 0.8. The atmospheric density considering the difference in altitude is written as

\[
\rho = \rho_0 + \Delta h \cdot \Delta \rho,
\]

where \( \rho_0 \) is the atmospheric density at the center of mass and \( \Delta h \) is the altitude difference from the center of mass. The aerodynamic torque is

\[
dT = \mathbf{r} \times (\delta \cdot d\mathbf{f}),
\]

where \( \mathbf{r} \) is the body-fixed coordinate vector of the unit surface element and \( \delta \) is the correction coefficient of the force that we add to account for unknown effects (See Sect. 3.3).
3.2 Orbit reduction

We define the time $t$ at UTC 0 h on Dec. 18, 2017 (mjd = 58105), as the origin. We fit the orbital elements from Nov. 18th to Mar. 8th. The semi-major axis and eccentricity, which are closely related to the orbit altitude, are reduced as follows.

$$a = -6.2463e^{-9} t^5 + 3.7859^{-7} t^4 - 8.503^{-6} t^3 - 0.0007853 t^2 - 0.345752 t + 6667.461 \text{ km},$$

$$e = 0.0018 + 0.0006 \sin(0.1305 t - 0.635).$$

The corresponding results are shown in Fig. 4. Other orbital elements are reduced as (unit: rad)

$$i = 0.745$$

$$\omega = 0.1289 t - 0.345$$

$$\Omega = 2.49 + \dot{\Omega} \cdot t$$

where $\dot{\Omega}$ is the linear precession of the orbital plane associated with the Earth’s oblateness, which is taken as -0.110 rad/day for Tiangong-1.

![Fig. 4. Fitting of the semi-major axis (left) and eccentricity (right) of Tiangong-1. The red dots are the instantaneous semi-major axis minus the radius of the Earth and eccentricity. The green lines are the fitting results. The small dots in the left panel are the mean semi-major axis values.](image)

3.3 Numerical results

In the simulation, only the molecular–surface interactions and gravity gradient torque are taken into account. The results are shown in Fig. 5. The blue line is calculated using the classical aerodynamic torque (regardless of the atmospheric density gradient), which yields a decreasing rotational speed. The brown and cyan lines consider the ADGT, where $\delta$ is set to 1, and they overcome the deceleration effect of the classic aerodynamic torque but do not match the measured data. The red line that is similar to the measured data is obtained when the coefficient $\delta = 3$, i.e., the force (and torque) is increased by a factor of 3.
Fig. 5. [1] Simulation results for the ADGT. The black points are the estimated rotational speeds (unit: °/s). The four curves represent the numerical simulations of the rotational speed. The blue line represents the results for the classic aerodynamic torque, and the remaining three lines consider the effect of ADGT, where the cyan and brown lines show the results when the solar panel is flat and upright relative to the main body, respectively, and the red shows the results when the atmospheric force acting on the model element is increased threefold when the solar panels are upright.

4 CONCLUSION AND DISCUSSION

The above analysis shows that the rotational acceleration of Tiangong-1 is closely related to the atmospheric density. If we assume that the ADGT model is valid, then we need to explore the physical explanation of $\delta$ equal to 3. One possible scenario is that the way we calculate the atmospheric density gradient is not realistic. The atmospheric density model may have problems dealing with density differences at distances of 10 meters. Another possible situation is described in Ref. [1], in which other minor effects in the interaction model, such as multiple reflection or hydrodynamic effects, may need to be considered. In addition, in such a microscopic molecular force model, a macroscopic physical quantity, namely, the density, is introduced, which may yield a contradicting result if considered. In any case, we hope that more researchers from different fields will work together to explore these issues.

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6 REFERENCES