In-orbit fragmentation characterization and parent bodies identification by means of orbital distances

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ABSTRACT

Once a potential fragmentation event has been detected by the Space Surveillance Network of sensors, it is necessary to confirm and characterize it. Typically, the network is observing a number of fragments crossing their field of view and a fundamental step for the analysts is the identification of the parent body (or bodies) of the observed fragments. We propose a new approach to correlate fragments with known orbits to parent objects, using the definition of a suitable orbital similarity function, like it is usually done in the case of asteroid families and meteor streams identification. The method can be used both if a short time has passed from the instant of breakup and if a long time has already elapsed. Among the known orbital distances (D-criteria) defined in the literature, we have chosen some of them as suitable metrics to be used for the case of space debris orbiting around the Earth. Moreover, we also consider the Minimal Orbital Intersection Distance (MOID) between two orbits (that is the absolute minimum of the Euclidean distance between a point on the first orbit and a point on the second one) as a further valuable possibility.

The developed method was applied to a known past fragmentation event, using the TLE data of the real fragments, and to some specific cases of simulated fragmentations (both explosions and collisions). The performance of the different D-criteria has been evaluated and the benefits and issues related to each one are discussed.

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1 INTRODUCTION

Orbital similarity functions defined in the space of Keplerian coordinates are commonly used to find the origin of meteor streams and to investigate the common origin of two or more asteroids. The scientific literature on this subject is rich. Historically, the first of this kind of functions was the D-criterion introduced by Southworth and Hawkins in 1963 [10] with the objective to identify meteor streams. Afterwards, the same D-criterion was applied for the identification of asteroid families, using proper-elements (see for example [8]). Other similar criteria were introduced for meteor streams identification respectively by Drummond [2] and Jopek [6].

The above mentioned three criteria were considered to be used to study clouds of Earth orbiting objects, with the objective to identify their origin. In this context, we concentrated on the identification of the breakup time and of the parent body (or bodies) of a fragmentation detected by the Space Surveillance Network of sensors, given that the orbits of some of the fragments are known. The idea is that the fragmentation should have happened at the time corresponding to the minimum of the mutual orbital distances of the known fragments, which is our candidate breakup time. In the same way, to find the parent among the known satellite orbits, we search the catalog for the minimum of the distance of any satellite from the fragments at the time of breakup.

In addition to the D-criteria, we considered also the Minimal Orbital Intersection Distance (MOID) [3] and the nodal distance to measure the similarity between two orbits. The MOID gives the absolute minimum of the distance between two osculating ellipses, while the nodal distance is their distance along the intersection of the orbital planes.

We implemented five different algorithms, differing only in the orbital similarity function, and tested them on simulated and real fragmentation clouds. In Section 2 we recall the definitions of the five selected similarity functions. In Section 3 we describe the algorithm for the breakup time determination and the parent body identification. In Section 4 we report the results of the numerical tests.

2 ORBITAL DISTANCES

The starting point of the proposed approach is the selection of a suitable similarity function d in the space of orbital elements, giving the distance between two orbits. A selection of possible orbital distances (D-criteria) for the identification of asteroid pairs is given in [9]. We have chosen some of them as suitable distances for the case of space debris orbiting around the Earth. Moreover, we consider the nodal distance and the MOID, defined in [3], as further valuable possibilities to investigate.

The final selection comprises the following: the classical D-criterion of Southworth and Hawkins [10], denoted by D_{SH}; the modification of D_{SH} proposed by Drummond [1] in the form given in [9], denoted by D_D; the modification of D_{SH} proposed by Jopek [4], denoted by D_H; the MOID, as defined in [3], denoted by D_{MOID}; the nodal distance, denoted by D_{nodal}.

Let a, e, i, Ω, ω denote respectively the semimajor axis, the eccentricity, the inclination, the right ascension of the ascending node and the argument of perigee. All the above mentioned distances depend on these 5 Keplerian coordinates, so that $d=d(a, e, i, \Omega, \omega)$. In what follows, the subscripts A and B are the indices of the two fragments of which we are computing the distance. The letter q denotes the perigee distance, $q=a(1-e)$.

2.1 D-criteria

The distance D_{SH} is defined by the equality

$$D_{SH}^2 = (e_B - e_A)^2 + \frac{(q_B-q_A)^2}{R_{Earth}^2} + \left(2\sin\left(\frac{I_{BA}}{2}\right)\right)^2 + \left(\frac{e_B+e_A}{2}\right)^2 \left(2\sin\left(\frac{\pi_{BA}}{2}\right)\right)^2, \quad (1)$$

where I_{BA} is the angle between the orbital planes of the fragments and π_{BA} is the difference of the longitudes of perigees measured from the intersection of the orbits, starting from anyone of the mutual nodes. In order to maintain all the terms of the expression of D_{SH} dimensionless, in Eq. (*1*) the perigee distance q is divided by the Earth's radius R_{Earth}. The quantities I_{BA} and π_{BA} are computed using the following equations:

$$\left(2\sin\left(\frac{I_{BA}}{2}\right)\right)^2 = \left(2\sin\left(\frac{i_B-i_A}{2}\right)\right)^2 + \sin i_A \sin i_B \left(2\sin\left(\frac{\Omega_B-\Omega_A}{2}\right)\right)^2, \quad (2)$$

$$\pi_{BA} = \omega_B - \omega_A + 2\sin^{-1}(S_{BA}), \quad (3)$$

$$S_{BA} = \cos\left(\frac{i_B+i_A}{2}\right)\sin\left(\frac{\Omega_B-\Omega_A}{2}\right)\sec\frac{I_{BA}}{2}. \quad (4)$$

The distance D_D is defined by

$$D_D^2 = \left(\frac{e_B-e_A}{e_B+e_A}\right)^2 + \left(\frac{q_B-q_A}{q_B+q_A}\right)^2 + \left(\frac{I_{BA}}{180^\circ}\right)^2 + \left(\frac{e_B+e_A}{2}\right)^2 \left(\frac{\theta_{BA}}{180^\circ}\right)^2, \quad (5)$$

where θ_{BA} is the angle between the lines of the apsides of the two orbits. For this distance, following [9], the angle I_{BA} is computed as the angle between the angular momentum vectors $\boldsymbol{c_A}$ and $\boldsymbol{c_B}$. The angular momentum $\boldsymbol{c}$ is defined as $\boldsymbol{c} = \boldsymbol{r} \times \boldsymbol{v}$, being $\boldsymbol{r}$ and $\boldsymbol{v}$ the geocentric position and velocity of the fragment. Then

$$I_{BA} = \cos^{-1}\left(\frac{\boldsymbol{c_A}\cdot\boldsymbol{c_B}}{c_A c_B}\right), \quad (6)$$

where $c_A = |\boldsymbol{c_A}|$, $c_B = |\boldsymbol{c_B}|$. The angle θ_{BA} is derived from the scalar product of the Laplace-Lenz vectors $\boldsymbol{e_A}$ and $\boldsymbol{e_B}$, where the Laplace-Lenz vector $\boldsymbol{e}$ is defined as $\boldsymbol{e} = (\boldsymbol{v} \times \boldsymbol{c})/\mu - \boldsymbol{r}/r$. Then, defining $e_A = |\boldsymbol{e_A}|$, $e_B = |\boldsymbol{e_B}|$, we have

$$\theta_{BA} = \cos^{-1}\left(\frac{\boldsymbol{e_A}\cdot\boldsymbol{e_B}}{e_A e_B}\right). \tag{7}$$

The distance D_H is defined by

$$D_H^2 = (e_B - e_A)^2 + \left(\frac{q_B - q_A}{q_B + q_A}\right)^2 + \left(2\sin\left(\frac{I_{BA}}{2}\right)\right)^2 + \left(\frac{e_B + e_A}{2}\right)^2 \left(2\sin\left(\frac{\pi_{BA}}{2}\right)\right)^2. \tag{8}$$

2.2 Minimal orbital intersection distance

Given two Keplerian orbits, their MOID is defined as the absolute minimum of the Euclidean distance between a point on the first orbit and a point on the second one. The algorithm of [3] allows us to compute all the critical points of the squared distance and in particular to determine all the minima. The distance D_{MOID} is the smallest one among them.

The computation of the MOID, as it is currently implemented, is computationally expensive, and it reveals to be not suitable when we deal with many fragments, i.e. when we need to compute the mutual distances of hundreds of orbits or more, unless parallelization is exploited. A possible alternative to be investigated is the usage of another algorithm for the MOID computation, among the ones already available in the scientific literature, like for example the solutions proposed in [1], [4] and [11]. A C++ software implementing [1] is made available by the authors at https://sourceforge.net/projects/distlink/, and the Fortran code implementing [11] is freely available at http://moid.cbk.waw.pl/.

2.3 Nodal distance

The nodal distance D_{nodal} is defined as the distance between the osculating orbits, provided at the same epoch, along the line of the mutual nodes. In order to give the explicit equations to compute it, let $\boldsymbol{c_A}$, $\boldsymbol{c_B}$ be the angular momentum vectors of the two fragments and let $\boldsymbol{e_A}$, $\boldsymbol{e_B}$ be the Laplace-Lenz vectors. The direction of the line of the mutual nodes is defined using the vector $\boldsymbol{d}_{node}$ given by

$$\boldsymbol{d}_{node} = \boldsymbol{c_A} \times \boldsymbol{c_B}. \tag{9}$$

The vector $\boldsymbol{d}_{node}$ points towards the ascending node of the body B with respect to body A. In the case of two coplanar orbits we select their common line of the nodes to define the vector $\boldsymbol{d}_{node}$. Denote by ν_A, ν_B the true anomalies of the mutual nodes of the bodies, in the direction of the vector $\boldsymbol{d}_{node}$, so that

$$\cos\nu_A = \frac{\boldsymbol{e_A}\cdot\boldsymbol{d}_{node}}{e_A d_{node}}, \qquad \cos\nu_B = \frac{\boldsymbol{e_B}\cdot\boldsymbol{d}_{node}}{e_B d_{node}}, \tag{10}$$

where $d_{node} = |\boldsymbol{d}_{node}|$, $e_A = |\boldsymbol{e_A}|$, $e_B = |\boldsymbol{e_B}|$. The geocentric distances of the two nodal points of the orbit A are given by

$$r_{A,1} = \frac{c_A^2}{\mu(1 + e_A\cos\nu_A)}, \qquad r_{A,2} = \frac{c_A^2}{\mu(1 - e_A\cos\nu_A)}, \tag{11}$$

where $\mu = GM_{Earth}$ is the Earth's gravitational parameter. The geocentric distances of the mutual nodes of the orbit B are given by

$$r_{B,1} = \frac{c_B^2}{\mu(1+e_B \cos \nu_B)}, \qquad r_{B,2} = \frac{c_B^2}{\mu(1-e_B \cos \nu_B)}. \tag{12}$$

Then we have two nodal distances, given respectively by $d_1 = |r_{B,1} - r_{A,1}|$, $d_2 = |r_{B,2} - r_{A,2}|$. The distance D_{nodal} is the minimum between d_1 and d_2.

3 BREAKUP TIME DETERMINATION AND PARENT BODY IDENTIFICATION

Suppose to have a cloud of fragments with known orbits. We want to determine the satellite from which the fragments originated. We call it the *parent*. We consider the American USSTRATCOM TLE public catalog, available at https://www.space-track.org, as reference for the known orbits of the objects orbiting the Earth. We assume that the satellite we are searching for was in the catalog before the fragmentation event. It may be still in the catalog or have disappeared after the event.

We first propagate the orbits of the fragments to a common time t_0, for example, the mean epoch of the first determined orbits of the fragments. Let d denote the selected distance function. Given the orbits referred to a common epoch, we compute the mean $d_{mean}(t_0)$ of the mutual orbital distances between them. We need to select a time $t_{prev} < t_0$ located before the fragmentation event. In order to be sure that the selected time is before the breakup, we can even choose a time very far from the appearance of the first fragment in the catalog, for example two months before this date. The orbits are propagated backward to time t_{prev}. A step size Δt is selected and the mean distance $d_{mean}(t_k)$ is computed at each step $t_k = t_0 - k\Delta t$ until the epoch t_{prev} is reached. The minimum of the mean distance d_{mean} is searched for in the interval (t_{prev}, t_0). We select as candidate breakup time the time t_b that corresponds to the epoch at which the minimum of d_{mean} is reached.

Once we have computed a possible time of breakup, we proceed with the search of the parent among a set of candidates extracted from the global catalog available at the time t_1 corresponding to the last update before t_b. To be more conservative, we can choose the time t_1 to be a few days before t_b, since the true breakup time in general may differ even much (1-2 days) from t_b. The candidates are characterized imposing limits, inferred from the orbital elements of the cloud, to the perigee and apogee altitudes. Any candidate orbit is propagated forward to the time t_b. Using the selected distance function d, we determine the parent orbit as the one minimizing the mean orbital distance from the fragments at time t_b.

The outlined procedure and the corresponding software are currently under study and the details are subject to modifications. In particular, the testing activity is still ongoing and there is space for reducing the runtime of the implemented software by focusing on its optimization. Furthermore, the procedure is going to be extended, with the application of a correction to the first estimation t_b of the breakup time, determined exploiting the knowledge of the parent orbit.

4 NUMERICAL TESTS

We tested the procedure outlined in Section 3 with the distances defined in Section 2 and compared the results obtained with the different distance functions. We performed two kinds of tests. First, we applied the procedure on simulated data. We used the NASA EVOLVE 4.0 breakup model [4] to generate the clouds of fragments. The note by Krisko [7] was followed for the correct implementation of the model and the conservation of the mass. Two clouds were generated simulating an explosion and a collision in orbit. The orbits of the fragments were propagated forward for 30 days after the event. A Gaussian error was added both to the parent orbit and to the propagated orbits of the fragments. A set of orbits crossing the cloud region were extracted from the TLE catalog, imposing limits on the perigee and apogee altitudes. These orbits together with the true parent were used as the set where to search for the parent. The algorithm based on the minimization of the orbital distance was applied, using, one at a time, the 5 selected similarity functions defined in Section 2.

Finally, we tested the procedure directly on real data, using the orbital data of the ORBCOMM FM-16 cloud of fragments, extracted from the TLE catalog.

The Gaussian error added to the orbits of the simulated events was computed in the following way. A diagonal covariance matrix Γ was defined in the radial-transversal reference frame of the object, the RTW frame, giving the position and velocity accuracy in that frame. The same matrix Γ was used for any object. Random errors were generated, having a Gaussian distribution with zero mean and covariance Γ. These errors were applied to the orbits of the fragments and of the parent by rotating from the RTW reference frame to the equatorial frame and adding the error to the Cartesian state vector. We identified 4 test cases:

a. No error.
b. Error in position equal to 10, 30, 20 m respectively in the radial, transversal and binormal direction. Error in velocity equal to 0.1 m/s in every direction.
c. Error in position equal to 100, 300, 200 m respectively in the radial, transversal and binormal direction. Error in velocity equal to 1 m/s in every direction.
d. Error in position equal to 1, 3, 2 km respectively in the radial, transversal and binormal direction. Error in velocity equal to 10 m/s in every direction.

4.1 Simulated explosion

The mass of the exploded spacecraft is set equal to 1000 kg. The breakup time is t_b = 58765,70885 MJD. The Keplerian coordinates of the exploded spacecraft at time t_b are (distances in km, angles in degrees):

$$(a, e, I, \Omega, \omega, n) = (7800.0, 0.0003, 80.3, 24.0, 345.0, 32.0\ ) . \tag{13}$$

The simulated cloud contains a total of 249 fragments with diameter $L_c \geq 10$ cm, 14 of which have diameter $L_c \geq 50$ cm. In order to have a set of candidates for the identification of the parent, a total of 3659 orbits were extracted from the TLE catalog imposing to have perigee distance smaller than 7900 km, apogee distance greater than 7500 km. The above limits were chosen on the basis of the orbital parameters of the fragments. The selected orbits together with the true parent given by Eq. 13 are the input candidates.

We performed 8 runs in total, 4 with the cloud of 249 fragments and 4 with the small cloud of 14 fragments. For each run, the algorithm for the identification of the parent was applied using, one at a time, all the distances selected in Section 2, except the MOID, which was used only for the small cloud. The 4 runs referred to the same cloud differ in the error applied to the orbits of the fragments and the parent and correspond to the 4 test cases defined at the beginning of Section 4. The behavior of the mean distances for the 4 test cases is shown in Figs. 1-2-3-4 for the big cloud, in Figs. 5-6-7-8 for the small cloud. The legend is the same for all the figures and is reported in Fig. 9. In all the figures the distances are normalized dividing by their value at the initial epoch t_0 of the cloud. From these figures it is evident that, a part small oscillations, the functions $D_{\_SH}$ and $D_{\_H}$ follow a parabolic curve with its minimum around the breakup time. The function $D_{\_D}$ has wider oscillations and follows an almost horizontal line. The functions $D_{\_nodal}$ and $D_{\_MOID}$ sink towards zero in correspondence of the breakup time, for the test case (a), but this feature is less and less marked increasing the error in the orbits.

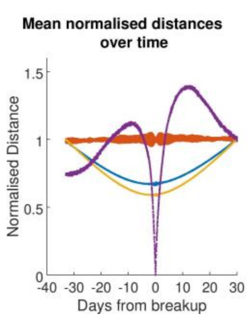

Fig. 1. Mean distances over time for the cloud of 249 fragments. Test case (a).

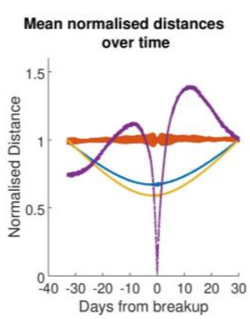

Fig. 2. Mean distances over time for the cloud of 249 fragments. Test case (b).

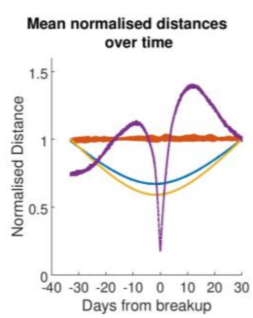

Fig. 3. Mean distances over time for the cloud of 249 fragments. Test case (c).

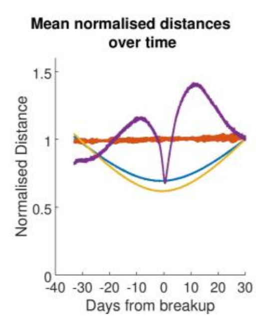

Fig. 4. Mean distances over time for the cloud of 249 fragments. Test case (d).

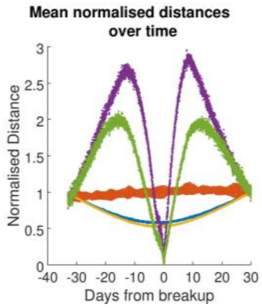

Fig. 5. Mean distances over time for the cloud of 14 fragments. Test case (a).

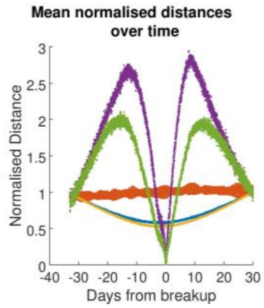

Fig. 6. Mean distances over time for the cloud of 14 fragments. Test case (b).

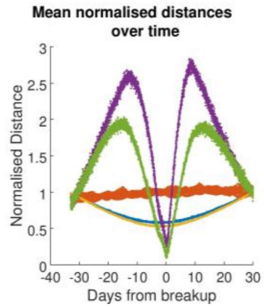

Fig. 7, Mean distances over time for the cloud of 14 fragments. Test case (c).

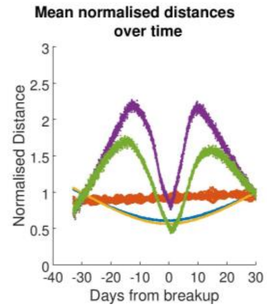

Fig. 8. Mean distances over time for the cloud of 14 fragments. Test case (d).

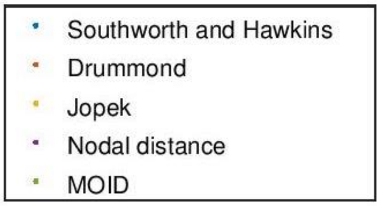

Fig. 9. Legend of Figs. 1-8 and Figs. 10-14.

We obtained similar results using the big and the small cloud, as it is evident by the comparison of Table 1 and Table 2. Among the D-criteria, the distance D_D reveals to be not reliable. This is due to the first term of Eq. 5 involving the eccentricity. Indeed, the orbits involved in this test have low eccentricity, as it frequently happens in the LEO regime. Consequently, this term is always high and dominates the other ones, even when the difference of the eccentricities is small. The distances D_{SH} and D_H are the most suitable for the identification of the parent. The effect of having less accurate orbits is not strong in the determination of the breakup time and the right parent is always identified. For the nodal distance instead, having less accurate orbits causes the algorithm to fail in the identification of the right parent, even if the determination of the breakup time is good. In general, using the nodal distance the determination of the breakup time is better than with the D-criteria, but there are remarkable exceptions. In order to have a reasonable runtime, the MOID was used only for the small cloud. The algorithm using the MOID failed in determining the parent only for big errors in the orbits. It allowed us to constrain the breakup time better than with the other distances, without exceptions.

Table 1. Runs with the explosion cloud of 249 fragments with diameter greater than 10 cm. The blue background means that the algorithm succeeded in determining both the breakup time and the parent. The yellow color means that only the breakup time was determined. The white color means complete failure.

Pos. Error in RTW (m)	Velocity Error (m/s)	Distance	Parent Found (Y/N)	Breakup time t_b (MJD)	Error in t_b (days)
0	0	D_{SH}	Y	58765.443650	0.2652
		D_D	N	58764.220050	1.4888
		D_H	Y	58765.965850	0.2570
		D_{nodal}	Y	58765.708950	0.0001
10x30x20	0.1	D_{SH}	Y	58765.443650	0.2652
		D_D	N	58764.219350	1.4895
		D_H	Y	58765.886050	0.1772

		D_{nodal}	Y	58765.704750	0.0041
100x200x300	1	D_{SH}	Y	58765.524850	0.1840
		D_D	N	58731.072250	34.6366
		D_H	Y	58764.806650	0.9022
		D_{nodal}	N	58765.713150	0.0043
$1x3x2 \cdot 10^3$	10	D_{SH}	Y	58763.951950	1.7569
		D_D	N	58736.213750	30.5049
		D_H	Y	58765.844750	0.1359
		D_{nodal}	N	58766.164650	0.4558

Table 2. Runs with the explosion cloud of 14 fragments with diameter greater than 50 cm. The blue background means that the algorithm succeeded in determining both the breakup time and the parent. The yellow color means that only the breakup time was determined. The white color means complete failure.

Pos. Error in RTW (m)	**Velocity Error (m/s)**	**Distance**	**Parent Found (Y/N)**	**Breakup time t_b (MJD)**	**Error in t_b (days)**
0	0	D_{SH}	Y	58765.521350	0.1875
		D_D	N	58730.638250	35.0706
		D_H	Y	58764.728250	0.9896
		D_{nodal}	Y	58765.708950	0.0001
		D_{MOID}	Y	58765.708950	0.0001
10x30x20	0.1	D_{SH}	Y	58765.521350	0.1875
		D_D	N	58736.931250	28.7776
		D_H	Y	58764.728250	0.9806
		D_{nodal}	Y	58765.711050	0.0022
		D_{MOID}	Y	58765.706850	0.0020
100x200x300	1.0	D_{SH}	Y	58764.712850	0.9960
		D_D	N	58735.261750	30.4471
		D_H	Y	58764.738050	0.9708
		D_{nodal}	N	58766.020450	0.3116
		D_{MOID}	Y	58766.019750	0.3109
$1x3x2 \cdot 10^3$	10	D_{SH}	Y	58762.848050	2.8608
		D_D	N	58739.111750	26.5971
		D_H	Y	58765.686550	0.0223
		D_{nodal}	N	58730.247650	35.4612
		D_{MOID}	N	58766.253550	0.5447

4.2 Simulated collision

The mass of the target spacecraft is 1000 kg, the mass of the projectile is 10 kg. The relative velocity of the projectile is 10 km/s. The breakup time is t_b = 58765.70885 MJD. The orbit of the target is the same as the one used for the simulated explosion, given in Eq. 13. The simulated cloud contains 55 fragments with diameter $L_c \geq 50$ cm. This is the set used as input for the tests. The same set of 3659 candidates, extracted from the TLE catalog, used for the simulated explosion, was used also for this simulation for the identification of the parent.

We performed 4 runs, corresponding to the 4 test cases defined at the beginning of Section 4. For each run, the algorithm for the identification of the parent was applied using, one at a time, all the distances selected in Section 2. The behavior of the mean distances for the 4 test cases is shown in Figs. 10-11-12-13, for which the legend is still the one reported in Fig. 9. The results are reported in Table 3. They confirm the conclusions obtained with the simulated explosion.

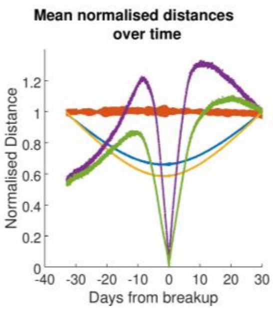

Fig. 10. Mean distances over time for the collision cloud. Test case (a).

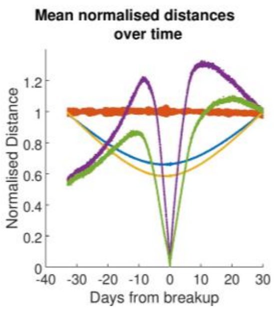

Fig. 11. Mean distances over time for the collision cloud. Test case (b).

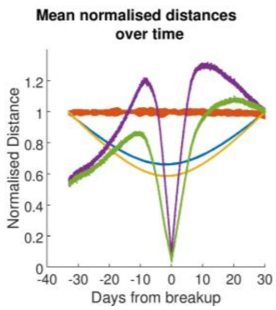

Fig. 12. Mean distances over time for the collision cloud. Test case (c).

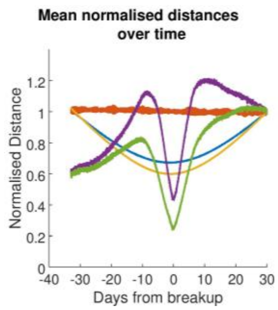

Fig. 13. Mean distances over time for the collision cloud. Test case (d).

Table 3. Runs with the collision cloud. The blue background means that the algorithm succeeded in determining both the breakup time and the parent. The yellow color means that only the breakup time was determined. The white color means complete failure.

Position Error in RTW (m)	**Velocity Error (m/s)**	**Distance**	**Parent Found (Y/N)**	**Breakup time t_b (MJD)**	**Error in t_b (days)**
0	0	D_{SH}	Y	58765.809750	0.1009
		D_D	N	58795.645150	29.9363
		D_H	Y	58765.808350	0.9005
		D_{nodal}	Y	58765.708950	0.0001
		D_{MOID}	Y	58765.708950	0.0001
10x30x20	0.1	D_{SH}	Y	58765.809750	0.1009
		D_D	N	58795.645150	29.9363
		D_H	Y	58765.808350	0.9005
		D_{nodal}	Y	58765.705450	0.0034
		D_{MOID}	Y	58765.705450	0.0034
100x200x300	1.0	D_{SH}	Y	58763.990450	1.7184
		D_D	N	58795.645850	34.937
		D_H	Y	58763.429750	2.2791
		D_{nodal}	Y	58765.706150	0.0027
		D_{MOID}	Y	58765.706850	0.0020
$1x3x2 \cdot 10^3$	10	D_{SH}	Y	58764.112250	1.5966
		D_D	N	58788.722150	23.0133
		D_H	Y	58765.063550	0.6453
		D_{nodal}	N	58765.849650	0.1408
		D_{MOID}	Y	58765.266550	0.4423

4.3 ORBCOMM FM-16 fragmentation

The ORBCOMM FM-16 spacecraft fragmented on December 22, 2018 (58474 MJD). The spacecraft has USSTRATCOM-SSN catalog number 25417. Only 9 fragments were in the TLE catalog on January 28, 2019.

Two sets of candidates for the identification of the parent were extracted from the TLE catalog. The first set counts 4382 orbits, which were selected imposing to have perigee distance greater than 7100 km, apogee distance smaller than 7400 km. A larger set was then considered, imposing to have perigee distance smaller than 7400 km and apogee distance greater than 7100 km. The above limits were chosen on the basis of the orbital parameters of the 9 fragments. The selected orbits include the ORBCOMM FM-16.

We performed 2 runs, corresponding to the 2 set of candidates. For each run, the algorithm for the identification of the parent was applied using, one at a time, all the distances selected in Section 2. The behavior of the mean distances over time is shown in Fig. 14. The results of the two tests are reported in Table 4. The distances D_{SH} and D_H are confirmed as the right ones for both the identification of the parent body and the first estimation of the breakup time.

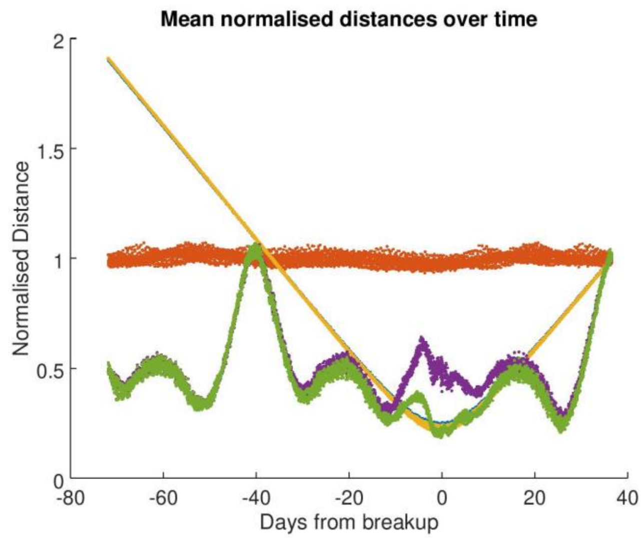

Fig. 14. Mean distances of the ORBCOMM FM-16 fragments over time. The legend is shown in Fig. 9. The distances are normalized dividing by their value at the initial epoch t_0 of the cloud.

Table 4. Results for the ORBCOMM FM-16 fragments. The blue background means that the algorithm succeeded in determining both the breakup time and the parent. The yellow color means that only the breakup time was determined. The orange color means that only the parent was determined. The white color means complete failure.

Candidates	Distance	Parent Found (Y/N)	Breakup time t_b (MJD)
SET A: 4382 orbits	D_{SH}	Y	58474.504274
	D_D	N	58470.288874
	D_H	Y	58474.434274
	D_{nodal}	Y	58499.625874
	D_{MOID}	N	58473.669874
SET B: 9157 orbits	D_{SH}	Y	58474.504274
	D_D	N	58470.288874
	D_H	Y	58474.434274
	D_{nodal}	N	58499.625874
	D_{MOID}	N	58473.669874

5 CONCLUSIONS

Different algorithms, based on orbital similarity functions, were implemented and tested on simulated and real fragmentation clouds. The distance criterion by Southworth and Hawkins and the one proposed by Jopek proved to be the most suitable ones for our purposes. Indeed, contrary to the other functions, they allowed us to find both the right breakup time and the right parent body in all the tested cases. Nonetheless, the availability of other criteria improves our capability to handle and characterize different types of events.

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