ESTIMATING BOTH IMPACT AND TSUNAMI CONDITIONS FOR A POSSIBLE COLLISION BETWEEN THE EARTH AND ONE APOPHIS-SIZED ASTEROID.
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Introduction: The topic of interest of this work is to numerically show the impact event with an object of dimensions similar to Apophis (we are not talking here about the same asteroid Apophis), which in April 2029 will pass within ~ 36,700 km (5.7 Earth radii) of the Earth, geosynchronous orbit (~ 36,000 km) [1]. Then, the aim of this work is to make only theoretical estimates based on some equations postulated by Holsapple (crater depth) [2], scaling, polynomial analysis, an adaptation of quantum formalism for the mathematical representation of the energy pulse that arise in the impact point, in where besides is used one solution (soliton type) of the Korteweg-DeVries’ equation [3], [6]. For the case of a tsunami due to one possible impact in the Pacific Ocean, the Ward and Asphaug model is used, [4], [7].

Analytical Method and Results: According to the models already described, the main variables for a possible impact of an asteroid similar to Apophis with the Earth would be the following: density of impactor ~ 2.38 g/cm³; mass of asteroid ~ 53.43 millions of tons; velocity of impact ~ 18.19 km/s; impact angle ~ 46.81°; energy of impact ~ 8.83×10²⁵ Erg (~ 2.10×10⁹ megatons). Some numerical estimates were contrasted (for reference only) with the impact calculator of H. J. Melosh and R. A. Beyer, Purdue University [5]. Using these same impact variables for a collision on the Pacific Ocean, and entering the Ward and Asphaug model, we will obtain a tsunami with the following characteristics, then, considering $R_p=2.05 R_C$ and $k_{max}=2\pi /2.11 R_C$, where $R_C$ and $R_p$ are inner and outer radii of the impactor, it is possible to estimate the velocity for the peak amplitude tsunami as [4], [7], $u_i(k_{max})=\varepsilon(k_{max})[0.5 + k_{max} \sinh(2k_{max})] = 194.98 \text{ km/h}$, where, $\varepsilon(k_{max})=\sqrt{\tan(k_{max})k_{max}}[\sqrt{5}, v_i=(gh)^{0.5}$ and $h=\text{average ocean depth} \approx 4.28 \text{ km}$. The frequency associated to this peak amplitude tsunami is expressed as [4], [7], $\omega_{max} = g k_{max} \tanh(k_{max})^{0.5} \approx 9.14 \times 10^{-2} \text{ Hertz}$, according these results, the period $t_{max}$ and wavelength $\lambda_{max}$ associated to this peak amplitude are estimated in ~ 10.95 s and ~ 592.85 m, respectively. The tsunami energy is estimated as [4], [7], $E_T = (1/3)\mu g \rho w D_c R_C^2 \approx 5.33 \times 10^{17} \text{ J}$, where, $\rho w \approx 1.027 \text{ kg/m}^3$ and $D_c$ is the initial cavity depth. The transverse diameter of the cavity [4], [7], is expressed according,

$$d_c=2R_d(2e V_f /g R_d)^{\beta}(\rho_w/\rho_0)^{\alpha /3}(1/q R_d^4)^{\beta /3} \approx 14.41 \text{ km},$$

being $R_d$, $V_f$ and $\rho_0$ impactor radius, impactor velocity and impactor density, respectively. Besides, laboratory adjustments [4], [7], are realized for the parameters $\alpha$, $\beta=1/2(1-\alpha)$ and $q$ as 1.27, 0.22 and 0.0077 respectively. Where $e$ is the fraction of impactor kinetic energy that transfers to the tsunami, then $e=(E_d/E_l)$, i.e., $e=(5.33 \times 10^{17} \text{ J} / 8.83 \times 10^{18} \text{ J}) \approx 0.06$, and $e^2 \approx 0.22 \approx 0.54$. The maximum tsunami height [4], [7], is,

$$u_{z_{max}}(r,R_c) = D_c [1/(1+r/R_c)]^{0.5} \left[ \left( \varepsilon_1 - 0.5 \right) \varepsilon_2 \right],$$

in where $\varepsilon_1 \approx 1.075$ and $\varepsilon_2 \approx 0.035$. In this formula, $u_{z_{max}}(r,R_c) \approx 974.48 \text{ m/s}$ if $r = R_c$, $u_{z_{max}}(r,R_c) \approx 798.40 \text{ m/s}$ if $r = 2.5 R_C$, and $u_{z_{max}}(r,R_c) \approx 103.02 \text{ m/s}$ if $r = (2.5 R_C + 50 \text{ km})$, where $r$ is the distance from the source or point of impact in the ocean.

Discussion: The main importance of the numerical values estimated here is to roughly quantify the momentum with which one asteroid similar to Apophis will approach Earth. This momentum is calculated as follows, $P_i = m_i V_i = (53.43 \times 10^5 \text{ kg}) (18.19 \text{ km/s}) = 971.89 \times 10^{12} \text{ kg} \text{m/s}$, where $P_i$, $m_i$ and $V_i$ are momentum of the impactor, mass of the impactor and velocity of the impactor, respectively. Then, this is the approximate amount of movement that must be considered, if we want to divert the trajectory of one asteroid similar to Apophis in a distant but possible scenario of impact with the Earth in the future. All this is hypothetical but based on solid foundations, and is part of the need to be prepared for an event of this type, not only with Apophis but with any object with similar characteristics. Finally, we can add that even though it is a hypothetical situation, it is helpful with regard to how to quantify the impact variables and a possible associated tsunami.