A Pre-Entry Shape Estimation for Puerto Lápice and Villalbeto de la Peña Meteorites via Statistical Distribution of Fragment Masses

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Introduction: One of the challenging physical problems is the estimation of the pre-entry shape of a meteoroid, since the detailed hypersonic body interaction with an atmosphere is unobservable directly. The results of the interaction are observable and measurable in the form of fragment masses. We use the approach to estimate the shape is based on the statistical mass distributions of the recovered. The power law with exponential cutoff is fitted to the empirical fragment distribution function to obtain the scaling index. An empirical quadratic equation relates the scaling index to the shape parameter, expressing the proportions of the of the initial unfragmented body. The technique for estimating the initial shape of a disintegrating body is based on the results of experiments on brittle fragmentation with the size of fragments being smaller than or equal to the minimum linear dimension of the body [1].

Solution Methods: The discrete distribution of fragment masses $\{m_i\}_{i=1}^{N}$ is approximated by a power law with exponential cutoff

$F(m,\beta_0,j,m_U) = \frac{N-j}{m_j}\left(\frac{m}{m_j}\right)^{-\beta_0}\exp\left(-\frac{m-m_j}{m_U}\right)$, where β_0 is a scaling index, m_j is the lower limit of sample completeness, and $m_U > m_j$ is the upper exponential cutoff mass.

The scaling index β_0 and the upper cut-off mass m_U are obtained by fitting to the known fragment distribution via the least squares method:

$$S(\beta_0,j,m_U) = \sum_{i=j}^{N}\left[\frac{N-j}{m_j}\left(\frac{m_i}{m_j}\right)^{-\beta_0}\exp\left(-\frac{m_i-m_j}{m_U}\right) - \frac{N-i}{m_i}\right]^2$$

The least squares are solved numerically find β_0^* and m_U^* that correspond to $\frac{\partial S}{\partial \beta_0} = 0$ and $\frac{\partial S}{\partial m_U} = 0$. The numerical algorithm combines Newton iterations $\beta_0^{(k+1)} = \beta_0^{(k)} - \left(\frac{\partial S}{\partial \beta_0} / \frac{\partial^2 S}{\partial \beta^2}\right)\Big|_{\left(\beta_0^{(k)},j,m_U\right)}$ and dichotomy for $m_U^{(\ell+1)}$ with range selection depending on the signs of the $\frac{\partial S}{\partial m_U}$.

We use the modified K-S norm [4] to find j^* delivering $D(\beta_0^*,i,j^*,m_U^*) \to \min$:

$$D(\beta_0^*,j,m_U^*) = \max_{m_i \ge m_j} \frac{\left|F(m_i,\beta_0^*,j,m_U^*) - \frac{N-i}{m_i}\right|}{\sqrt{F(m_i,\beta_0^*,j,m_U^*)\left[\frac{N-i}{m_i} - F(m_i,\beta_0^*,j,m_U^*)\right]}}$$

The obtained parameter β_0 is used in the empirical equation of the second order $0.13d_m^2 - 0.21d_m + (1.1-\beta_0) = 0$.
The dimensionless parameter d_m is related to the meteorite linear proportions a, b, c as $d_m = 1 + \frac{2(ab+ac+bc)}{a^2+b^2+c^2}$.

Results: We compare the meteorite pre-entry shape estimated as linear proportions against the respective counterparts of Košice, Bassikounou [2] and Sutter's Mill [3] meteorites.

Meteoroid	β_0	m_j, g	m_U, g	d_m	a/c	b/c
Košice	1.53	5.64	155.17	2.8	2.0	1.69
Bassikounou	1.32	29.9	2839.42	2.34	2.98	1.13
Sutter's Mill	1.51	5.0	21	2.76	2.13	1.58
Puerto Lápice	1.73	4.5	4.98	3.14		
Villalbeto de la Peña	1.61	52.8	518.13	2.95		

References: [1] Oddershede L. et al. (1993) Physical Review Letters. 71:3107. [2] Vinnikov V. et al. (2015) Proceedings IAU Symposium. 306. [3] Vinnikov V. et al. (2014) 45th Lunar and Planetary Science Conference. [4] Clauset A. et al. (2009) SIAM Rev. 51, 661–703.