Transport in the Lunar Water Exosphere. Norbert Schörgofer\textsuperscript{1,2}, \textsuperscript{1}Planetary Science Institute, Tucson, AZ and Honolulu, HI, USA, \textsuperscript{2}TREX-SSERVI (norbert@psi.edu)

Introduction: The surface-bounded exosphere of the Moon [1, 2] consists of ballistic trajectories, and is governed by its interaction with the surface. A water exosphere can transport molecules to cold traps near the rotational poles where they condense into ice [3].

When a molecule comes in contact with the surface, it thermally accommodates and then leaves in a random direction and with a thermal velocity distribution. A thermal ballistic hop of an \( \text{H}_2\text{O} \) molecule spans on average 200 km and takes 7 min.

Vertical density profile: Exospheres above an atmosphere, such as on Earth and the Sun, have long been investigated theoretically [4–6]. For the simple case of constant gravitational acceleration \( g \), these investigators obtain an exponential density distribution

\[
\rho(z) = \frac{\sigma}{H} e^{-z/H}
\]  
(1)

where \( \sigma \) is the column-integrated mass, \( z \) the height above the surface (or exobase), and \( H \) the scale height \( H = k_B T / (m g) \). Here, \( k_B \) is the Boltzmann constant, \( T \) the surface temperature, and \( m \) the mass of the molecule or atom.

These calculations assume a reservoir of molecules at the base. This situation applies for the water exosphere of a fully ice covered body, but on the Moon the number of available molecules is limited. The vertical density distribution for a fixed number of particles is significantly different from (1).

Given a probability distribution of initial velocities, the ensemble average of a quantity per hop/particle is denoted by \( \langle \rangle \). The average of a quantity at a given time, denoted by \( \langle \rangle_t \), has to be weighted by the flight duration. This is the time average of a stationary situation, or, with enough particles, a snapshot. For example, the average maximum height of a ballistic hop is \( \langle z_{\text{max}} \rangle = H/2 \) and \( \langle z_{\text{max}}^2 \rangle = H^2/3 \).

The vertical density profile of a stationary thermalized surface-bounded exosphere in a uniform gravity field, with a Maxwell distribution for launch velocities is

\[
\langle \rho \rangle(z) = \frac{\sigma}{2H} e^{-z^2/2H} K_0 \left( \frac{z}{2H} \right)
\]  
(2)

where \( K_0 \) is the modified Bessel function of the second kind. Figure 1 shows \( \langle \rho \rangle \) compared to the barometric formula (1). The density approaches infinity near the surface and decays faster than exponential at great height.

Surface residence times: The constituents of the lunar exosphere can be condensable (e.g. \( \text{H}_2\text{O} \)) or non-condensable (e.g. Ne, He). Basic statistical mechanics provides relations between the saturation vapor pressure \( p_v \), the sublimation rate into vacuum \( E \), and the residence time \( \tau \):

\[
E = \frac{p_v}{\sqrt{2 \pi k_B T m}}
\]  
(3)

\[
\frac{1}{\tau} = \frac{\alpha E}{\theta_m}
\]  
(4)

\[
p_v \propto \exp \left\{ -\frac{1}{k_B T} \left[ (\text{Enthalpy}) - (\text{Entropy}) \right] \right\}
\]  
(5)

The saturation vapor pressure is defined by a balance between the number of outgoing and incoming molecules. It provides the sublimation rate, even if there are no incoming molecules (3). The average of the inverse of the residence time is the sublimation rate per (exposed) particle (4); \( \theta_m \) is the number of molecules per area for a monolayer, and \( \alpha \) is the condensation coefficient. Knowledge of the (difference in) sublimation/desorption enthalpy alone does not provide the residence time, because the (difference in) entropy is also required (5). On a surface with defects, the entropy differs from that on a perfect crystal.

Although desorption energy measurements have been made for lunar soils and analogues [e.g., 7], extensive measurements of the molecular residence times of water molecules are lacking. Adsorption isotherms [8] can be used to determine the residence time of water molecules adsorbed on the grain surface. Measurements are reproduced in Figure 2 and approximated by an empirical fit. For a reversible isotherm, residence times can be calculated as a function of \( \theta \) via eqs. (3) and (4).

The surface residence times are relevant for exosphere modeling as well as for near-surface diffusion.
Some important numerical results: The ballistic hops in an exosphere can be readily modeled numerically, either by integrating the trajectory numerically or by computing the landing position analytically. There are implementations by Butler, Moore, Kegerreis, Schorghofer [11], Prem, and probably others.

Dusk-dawn asymmetry. For condensible species, such as H$_2$O, the density of the exosphere and the surface concentration vary greatly with local time. Exosphere-supplied water is concentrated around the morning terminator, but not around the evening terminator, a strong dusk-dawn asymmetry [12, 13].

Capture. Since the hop length is significantly larger than any cold trap, the water exosphere supplies the cold traps uniformly. There is a dependence on the distance from the source, increasing distance leads to more photo-destruction—, but otherwise deposition is uniform [12]. The “rain shadow” effect is negligible. (The strong latitude dependence found in Ref. [14] is not reproduced by other models.)

Capture rate. The fraction of exospheric water molecules that is trapped depends on the trapping area, which is so far only known in order of magnitude. The total cold trapping area on the Moon is much smaller than the area of permanent shadows, and it depends on the spatial resolution. Modern estimates [15] are significantly lower than the historical estimate of 0.5% of surface area [e.g., 3], which assumed half of all PSRs are cold traps. If the total trapping area is 0.1% of the surface area, about 10% of the available water is trapped before it is lost by photo-destruction at normal solar activity [12]. The trapping fractions on Mercury and Ceres are two orders of magnitude lower than the Moon’s (Figure 3), which amplifies the conundrum that the Moon has less exposed water ice than both of these bodies. This would make consistent sense, if the Moon had a much lower supply of water than either of these bodies.

Discussion: Remaining uncertainties in modeling of the lunar exosphere can be readily reduced with additional modeling (including cross-validation), laboratory measurements, and comparisons with observational data.

References