DETERMINATION OF LAVA TUBE DEPTH AND SHAPE FROM TOPOGRAPHY. E. A. Williams[1] and L. G. J. Montési[1]. [1]University of Maryland, College Park, MD, USA, ewilli17@umd.edu

Introduction: Lava tubes are long, often sinuous void spaces left inside some lava flows. They have been studied on Earth and probably exist on other planetary bodies, especially the Moon and Mars [1, 2]. Tubes and other caves would be practically useful and scientifically interesting locations for planetary exploration; they can protect anything inside them from surface hazards and provide well-preserved study sites to investigate a planet's volcanic history [3-5]. To facilitate the study of tubes on other bodies and plan how to use them on future missions, it is important to develop methods to evaluate the tubes' internal depths, shapes, and dimensions before direct exploration. Studying planetary tubes based solely on remote sensing data cannot reveal tube shape in detail. However, we show here that the height of a topographic ridge above inflated tubes can provide some constraints on tube dimensions. Ridge height is linked to the thickness of the roof and the height and width of the tube itself. With this relationship, observations of the surface above a tube could be used to learn about its internal structure.

Apparent Roof Thickness: Increasing the pressure inside a tube results in the development of an inflation ridge. If the tube roof has a constant thickness T, the ridge height is proportional to T^3 (Fig. 1a; laccolith model [6]). No formula exists for the ridge height above a tube with a more realistic, half-ellipse shape (Fig. 1b). Here, we develop a relationship that relates the shape parameters of a buried half-ellipse tube to an apparent roof thickness, T_a, which is the thickness of a laccolith roof that produces the same height ridge as the actual tube. The apparent roof thickness could be deduced from observations and provide constraints on the actual tube shape, parameterized by the central roof thickness T, central tube height c, and tube width W.

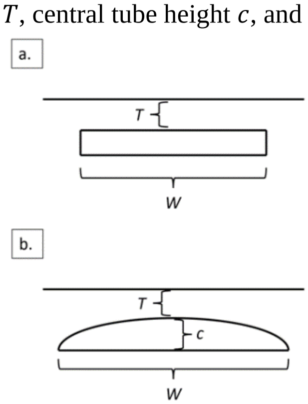

Figure 1: *Schematic of laccolith (a.) and half-ellipse (b.) tube shapes. T is the roof thickness above the center of the tube, c is the tube height at its center, and W is the full width of the tube.*

Methodology: We determined numerically inflation ridge height for a wide variety of tube dimensions using COMSOL© Multiphysics™. For each tube shape, we calculate the apparent roof thickness to the third power, T_a^3. The power of three is motivated by the analytical solution for a constant roof thickness. This quantity is then compared with the equivalent cubed roof thickness, which is the average of the spatially-varying roof thickness to the third power, $T_e^3 \equiv \frac{2}{W}\int_0^{W/2} t^3 dx$, where $t = T + c\left[1 - \sqrt{1 - \left(\frac{2x}{W}\right)^2}\right]$ is the roof thickness for an elliptical tube of height c, width W, and minimum roof thickness T. Fig. 2 shows that T_e^3 and T_a^3 are not equivalent, although they seem to be correlated to one another. Next, we develop an empirical relation between these quantities.

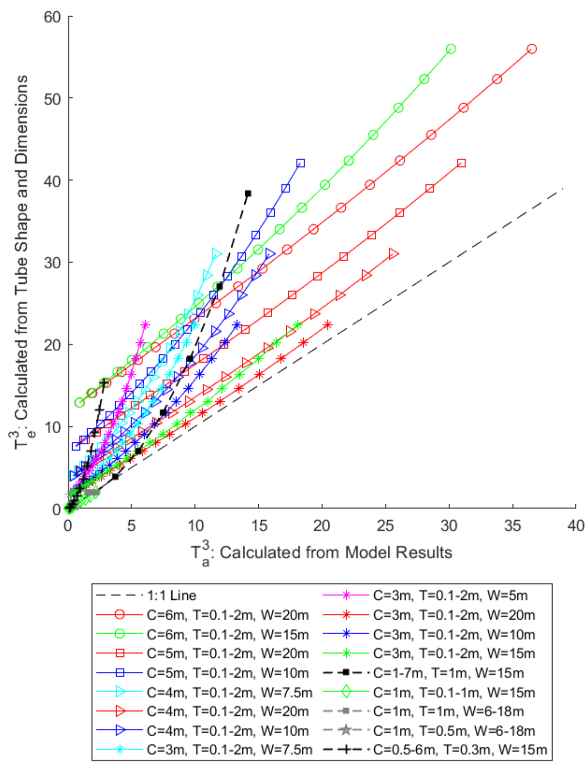

Figure 2: *Averaged roof thickness cubed, T_e^3, calculated from the tube shape, compared to the resulting apparent roof thickness T_a^3. The legend describes the model set up for each series.*

Quantifying the Dependence on Tube Height c. For a given tube width W, T_a^3 increases in proportion to tube height c. c does not influence the slopes of the lines in Figure 2, only their vertical positioning. Thus, we calculated for each c the offset required to match the data to the expected line and found it equal to $0.055\ c^3$. The power of 3 makes the coefficient of 0.055 dimensionless and, therefore, size-independent. Subtracting this offset from T_e^3 made all the lines in Figure 2 start from a single point; however, their slopes and curvature depend on tube width W.

Quantifying the Dependence on Tube Width W. The slope of each T_e^3 vs T_a^3 line depends on W, with little to no dependence on c. We examined the relations obtained for tubes of different widths and found that they were all fit well by the polynomial $Y = \frac{46}{W^3}X^2 + X$, where $Y = T_e^3 - b\ c^3$ is the equivalent cubed roof thickness corrected for tube height, and X is T_a^3. The coefficient of 46 is once again dimensionless and, therefore, independent of scale. Scaling this equation by tube width W, we obtain the final non-dimensional equation:

$$\tilde{T}_e^3 - b\ \tilde{c}^3 = \tilde{T}_a^3\left(1 + a\ \tilde{T}_a^3\right) \qquad \text{(Eq. 1)}$$

where $b = 0.055$, $a = 46$, and, a tilde denotes scaling by W. For completeness, $\tilde{T}_e \equiv \left[\frac{2}{W}\int_0^{\frac{W}{2}}\left(\frac{t}{W}\right)^3 dx\right]^{\frac{1}{3}}$.

Solving this equation gives an apparent roof thickness based only on the shape and dimensions of the tube and agrees with the model results. The comparison between shape-based and observed apparent roof thickness is shown in Fig. 3.

Application to Valentine Cave: The relationship in Eq. 1 can also be used to infer tube dimensions from observations of inflation ridge height. One example is Valentine Cave, in Lava Beds National Monument, California. This lava tube has been extensively studied with various geophysical techniques [7]. Of particular interest, LiDAR scans of the surface reveal a two-meter-high inflation ridge [8]. Previous numerical models of this cave using a Young's modulus, inflation pressure, and tube shape inspired by in-situ field observations give an apparent roof thickness cubed T_a^3 of 1.86 m^3 [9]; assuming the same tube shape, our relationship estimates a value of 1.93 m^3.

Eq. 1 could also be applied to observed tubes on the Moon and Mars, where remote sensing can reveal some but not all information about a tube. The width and height of an inflation ridge above a tube may be determined from high-resolution topographic data. Assuming the mechanical properties of the tube roof and the pressure responsible for inflation makes it possible to evaluate the apparent roof thickness. Then, Eq. 1 constrains the possible combinations of roof thickness and tube height. If either can be determined, thanks, for example, to imaging a skylight [10], the shape of the tube can be fully described.

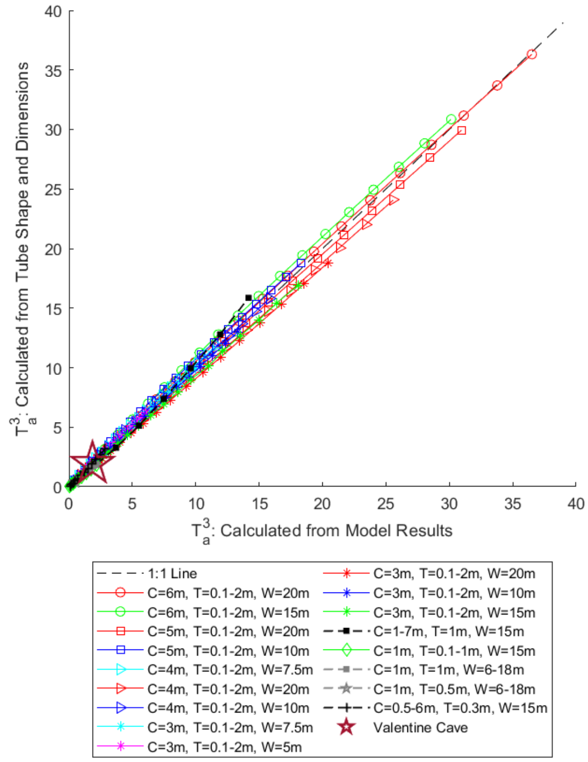

Figure 3: *Apparent roof thickness cubed, T_a^3 calculated using our Equation 1, compared to T_a^3 obtained in numerical models of tube inflation. The legend shows the dimensions used for each set of data. The purple star shows the data for Valentine Cave.*

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References: [1] Sauro F. et al. (2020) *ESR*, 10.1016/j.earscirev.2020.103288. [2] Wynne J.J. et al. (2022) *JGR: Planets* 10.1029/2022JE007303. [3] Hörz F. (1985) *Lunar Bases and Space Activities of the 21st Century* https://hdl.handle.net/20.500.11753/789 p.405-412. [4] Boston P.J. et al. (2004) *AIP Conf. Proc.* 10.1063/1.1649667. [5] Wynne J.J. et al. *JGR: Planets* (2022) 10.1029/2022JE007194 [6] Turcotte D. & Schubert G. (2014) *Geodynamics*. [7] Esmaelli S. et al. (2020) *JGR: Planets* 10.1029/2019JE006138 [8] Whelley P.L. et al. (2017) AGU, Abstr. T44D-04. [9] Williams E. A. et al. (2022) AGU, Abstr. V12C-0050. [10] Wagner R.V. & Robinson M.S. (2014) *Icarus* 10.1016/j.icarus.2014.04.002; Wagner R.V. & Robinson M.S. (2022) *JGR: Planets* 10.1029/2022JE007328