Examining Froude Number as a Scaling Factor for Granular Media in Changing Gravity using Dynamic Resistive Force Theory

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Abstract: Dynamic Resistive Force Theory is modified to study Froude number scaling for oblique granular impacts in varying gravity. Discrete simulations verify findings.

Introduction: Granular regolith strewn across the surfaces of planets, moons, and asteroids poses a challenge to the human-made craft that interact with it. The gravities of these far-away places is a major culprit, as changing gravity directly affects the ways in which granular media responds to perturbation.

This work explores the granular response to oblique surface impacts for gravities ranging from Earth-like to asteroid-like, utilizing a gravity of 6.27E-5 m/s^2 taken from Asteroid 101955 Bennu. It is shown that Froude number scaling for granular impacts can be extended into the oblique impact regime by using a slightly modified form of Dynamic Resistive Force Theory (DRFT). Froude number is defined as $\mathrm{Fr} = v/\sqrt{g\,r}$ where v is the projectile velocity, g is the local gravity, and r is the projectile radius [1]. The findings from DRFT are corroborated using LAMMPS discrete element simulations.

Methods

Dynamic Resistive Force Theory in space. Dynamic Resistive Force Theory (DRFT) [2] is a version of RFT [3] that simulates interactions with granular media by approximating resistive forces acting on the intruding body. It accounts for both the quasi-static and inertial contributions of resistance through the expression

$$t_{x,z} = \alpha_{x,z}(\beta,\gamma)\mathrm{H}(\tilde{z})|\tilde{z}| + \lambda\,\rho\,v_n^2\,n_{x,z} \quad (1)$$

where $\alpha_{x,z}$ is an empirically determined value tied to frictional resistance and lambda is a value that scales the inertial resistance. $\tilde{z}$, ρ, v_n, and $n_{x,z}$ are the depth of the intruder, the density of the media, the velocity of the intruder, and the surface normal of the intruding face.

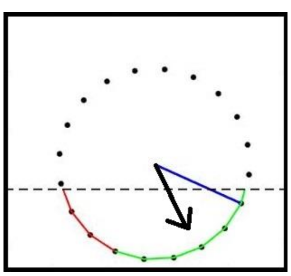

Figure 1: A frame from the DRFT code. Green segments mark regions where resistive forces are calculated. The black arrow shows the approximate direction of motion of the projectile.

A restriction of DRFT is that the empirical data is often experimentally sourced and only reflective of terrestrial environments. However, by making some small manipulations to Equation 1, it can be extended to other gravitational environments. In the second term on the right hand side, the quotient v_n^2 is rewritten in terms of Froude number by substituting $grF\mathrm{r}^2$ where r is the radius of the projectile and g is the local gravity. Local gravity can also be pulled out of the first term by multiplying g by the Earth-gravity (g_E) normalized version of $\alpha_{x,z}$ written as $\alpha_{x,z}^0 = \alpha_{x,z}/g_E$. Equation 1 now becomes

$$t_{x,z} = g\left(\alpha_{x,z}^0(\beta,\gamma)\mathrm{H}(\tilde{z})|\tilde{z}| + \lambda\rho r\mathrm{Fr}^2\,n_{x,z}\right) \quad (2)$$

Equation 2 is in a form that can be used with different gravitational conditions. This expression is implemented in Matlab to simulate the granular impact in two dimensions, where the projectile is modeled as a circle discretized into line segments. When in contact with the granular 'surface' (defined as the halfspace below z=0), these edges are used to compute resistive forces that are summed over the intruder face (see Figure 1). As time advances, these forces drive the projectile motion.

Discrete simulation setup: Discrete simulation in LAMMPS software serves to corroborate findings from DRFT. In these models, the granular media is defined as polydisperse soft spheres with properties inspired by glass (see Figure 2). The projectile is modeled with the same properties at a larger radius and launches into the granular bed with an angle between 20° and 70° and a velocity of 1 m/s to 7m/s for Earth gravity, giving a Froude number range of 3.55 to 24.87.

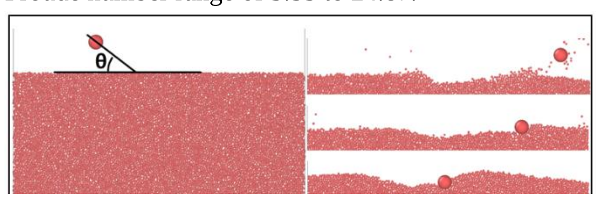

Figure 2: Initial setup for discrete impact tests with angle convention (left). Examples of ricochet/roll-out/full-stop behavior types (right).

For each impact case, three different outcomes may result, as classified by Wright et al. [1]. There are ricochets, where the projectile becomes airborne after departing the impact crater; roll-outs, where the projectile departs the crater but remains in contact with granular media; and full-stops, where the projectile remains within the impact crater (see Figure 2).

Results

DRFT simulation of oblique impacts. The numerical implementation of DRFT is used to model the impact of a projectile at 45° and a Froude number of 24.87. This impact produces a ricochet and is conducted in Earth, Moon, and Bennu gravities with time scales of 0.1s, 0.25s, and 39.6s, respectively.

The impacts in Figure 3 occur under Bennu (top), Moon (middle), and Earth (bottom) conditions. The projectile is shown for multiple frames, red lines denote current rotation, and yellow lines track the center of the projectile and its top-most and bottom-most points. Comparing the three plots in terms of trajectory and rotation over time, it is evident that the Froude number scaling yields identical results across gravitational conditions and that the gravity adjustable form of DRFT shown in Equation 2 works as intended.

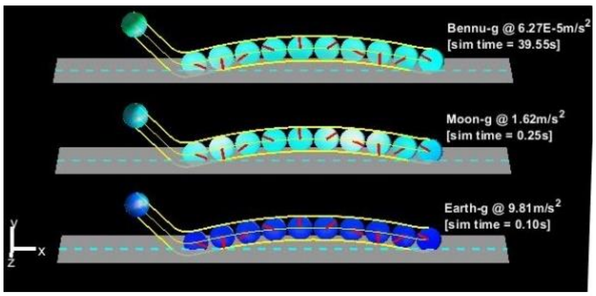

Figure 3: Results of DRFT numerical simulation for impacts at 45° and a Froude number of 24.87 for Bennu (top), Moon (middle), and Earth (bottom) gravity.

Corroboration with discrete simulations: The DRFT simulation suggests Froude scaling works in oblique impact cases, however corroboration with conventional discrete element simulation is warranted.

Using the LAMMPS model, impacts are simulated for every 5° and 0.5m/s increment (referencing Earth conditions). Also, two different projectile initial locations are used to reduce the effect of stochasticity from rough surface topography.

Under Earth gravity, the left contour plot of Figure 4 is produced. To create the map, ricochets are marked '2, white', roll-outs marked '1, red', and full-stops marked '0, black'. The blending of contours conveys the gradual transition of behaviors between each type.

The impact simulations are repeated at Moon gravity (1.62 m/s^2) and Bennu gravity (6.27E-5m/s^2). The classifications for the Moon and Bennu campaigns are shown in Figure 4 as well.

By qualitative comparison of Figure 4's plots, it is observed that the Froude number scaling is effective at preserving the impact outcomes across gravitational environments, even for oblique impact events. This expands upon the utility of Froude number scaling suggested by Sunday et al. [4], testing across larger Froude number and gravitational ranges as well.

Conclusion: Modification of DRFT's formulation demonstrates how Froude number ties into the very basis of granular mechanics. It is shown that DRFT can be rewritten to work for different gravitational environments without drastic overhaul of its empirical data. A numerical implementation of DRFT shows Froude number scaling does indeed collapse the projectile trajectory across gravitational conditions, even when considering an oblique impact event. The findings from DRFT code are verified using discrete simulations, which show on a larger scale Froude number's powerful ability to constrain the granular response when gravity changes.

Acknowledgements

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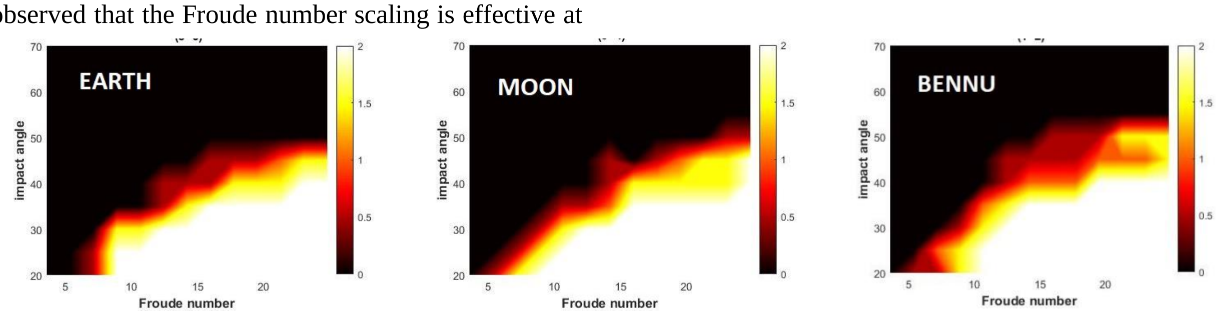

Figure 4: Impact behavior maps for Earth, Moon, and Bennu. White shows ricochets, red roll-outs, and black full-stops.