

HERTZIAN CONTACT THEORY FOR A MARTIAN CONTEXT. A. Bates¹, J. M. Lorenzo¹, S. Karunatilake¹
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Introduction: Prior workers have hypothesized the various states and stability regimes of regolith-bound H₂O-ice on Mars based on the diffusion of atmospheric water vapor into the porous martian regolith (1–4). Such models primarily use fluctuations in the orbital dynamics of Mars, especially obliquity variations, to determine periods of thermophysical stability of ice as a function of latitude. As Mars’ obliquity changes, the stability regimes for ice change, resulting in latitudinal transfer of ice through vapor diffusion (1, 5). However, ice may remain metastable at shallow depths within the mid-latitudes (~55°) even during low obliquity, as verified by satellite observations (6, 7). In addition to poorly constrained metastability, the quantity of ice found at lower latitudes exceeds what could have accumulated solely through available atmospheric vapor through the previous obliquity cycle (1, 6).

To understand seasonal controls and identify key regions of regolith ice accumulation, future in situ missions must capably investigate the martian subsurface. However, robust characterization of the subsurface is reliant upon a priori knowledge of the physical properties of the materials which constitute the regolith and any cementing material which may be present. In comparison to Earth, the martian regolith is largely chemically homogeneous due to mechanical erosion of the primary basaltic crust and subsequent lithification over geologic timescales (8). While chemical alteration is present on Mars and can certainly affect the composition and mechanical properties of the regolith, such instances of extreme chemical alteration are generally spatially limited and not representative of the composition of the martian surface as a whole (9, 10). In order to effectively investigate the martian subsurface for water ice, we must first constrain velocity magnitudes that could occur as the result of a dry, wet or cemented regolith.

Assessing the ability of established terrestrial methods of velocity estimation to function in a martian context is the primary goal of this work. As such, we model the seismic velocities (p- and s-wave) through a medium whose mechanical and chemical properties are similar to the bulk martian surface. For this approach, we compare the velocities estimated through contact theory with those measured in the lab (11) and those gathered by InSight (12). We find that simple contact theory is unsuitable for effective modelling of velocities on Mars. However, after adopting an empirical correction from prior work (13), we see considerably more overlap in velocity magnitude between our modeled velocities, InSight and lab measured velocities.

Methods: Hertzian contact theory (14) predicts the seismic velocity as a function of the contact forces

present between 2 or more grains. Velocity magnitude is dependent on the nature of the contact, the shape of the grain, and the physical properties of the grain(s). We therefore quantify the physical properties our soil grains based on the mineralogical data provided by (11). Using the Reuss lower bound, we calculate the overall effective elastic modulus of a system of grains whose composition is the aggregate of overall mineralogy as determined by (11). This provides a conservative estimate of the overall elastic modulus of a dry system of grains in contact with each other, similar to what is present at InSight (15). This estimation of elastic moduli is then used to model velocity through contact theory. Other input parameters include coordination number, a proxy for the number of grain contacts, for which we use 6, which is slightly less than typical for lightly packed spherical sand grains (16). In addition, we also use a composite density, simply an average density weighted by the volume amount of each mineralogical constituent, following the data from (11), with the addition of Earth atmosphere in the unoccupied pore space. Initial porosity of the system is set at 0.4, roughly equivalent to the initial porosity of each sample as determined by (11).

Comparing the contact theory estimated velocities with those measured in the lab by (11) shows that contact theory is unable to recreate the velocity profile of the medium as it would occur in nature (Fig. 1). We therefore employ an empirical correction following

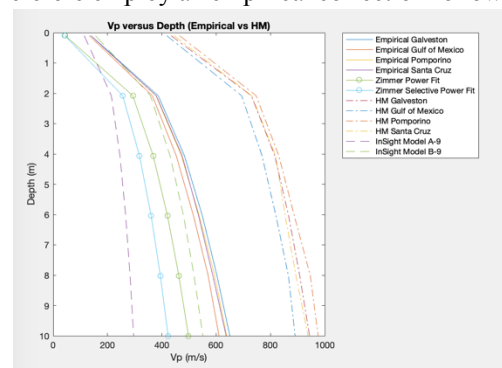


Figure 1: Hertzian contact theory (right most, dot-dashed) compared to its empirical correction (solid). The correction shows much greater similarity to the lab measurements upon which the predictions were based. equations 7-9 from (13). This correction combines the elastic moduli of the system into a single term α^2 , which can either be numerically determined or be replaced by an appropriate integer. We determined an appropriate integer (Fig. 2) by ratioing the velocities from various α

values with the lab measured velocities from (11) in order to determine where the ratio of the two values was nearly 1. This value, roughly 8100, represents the α value which most closely predicts a velocity similar to what was measured in the lab across the different low pressure intervals.

Results: The average velocity of the various sands measured by (11) exist within the bounds of possible velocity profiles at InSight, suggesting that the processes of compaction on Mars may create a shallow regolith structure which has a velocity magnitude similar to spherical sands here on Earth. However, Hertzian contact theory consistently overpredicts velocity magnitudes in comparison to their lab measured counterparts (Fig. 1). After applying an empirical correction, the overprediction is still present, but much less pronounced. Replacing α with a value of 8100, as shown in Fig. 3, places the velocity profile within the bounds of the two InSight profiles, and recreates much of the same geometry and velocity magnitudes as the lab measurements.

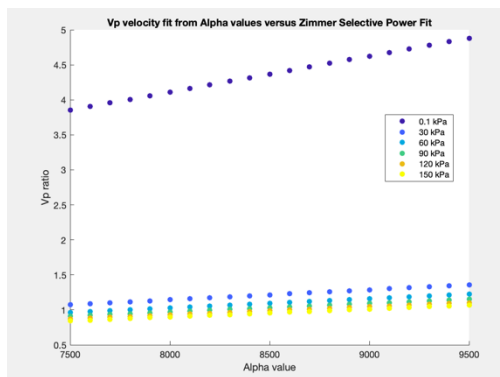


Figure 2: Ratio of lab measured velocity to the various α values (x-axis). The α value which corresponds to a near 1 ratio is ~ 8100 .

These results are consistent with past work (13), which find that Hertzian contact theory is unable to replicate the velocity profile of unconsolidated sands found in nature. Using a Reuss estimation of elastic moduli and applying an empirical correction, we are able to recreate the geometry and nearly match the upper bound regolith velocity magnitude at InSight.

Discussion: From this work, we can confidently conclude that Hertzian contact theory is unsuitable for forward modelling the possible velocity structures which may exist in the shallow martian subsurface. Where prior work attributed the Hertzian overprediction to possible clay content, we can also conclude that this is an unlikely culprit. Velocity comparisons between samples devoid of any clay (11) and Hertzian contact theory velocities failed to exhibit any similarity in velocity magnitude (Fig 1, 3). In addition, these samples

were devoid of any water, further suggesting that the differences in measured versus predicted velocity must be from another source. The functional form of the empirical correction from (13) is similar to that of Hertzian contact theory (14), implying that at shallow depths, grain contacts and lithostatic pressure have the most profound effect on seismic wave propagation.

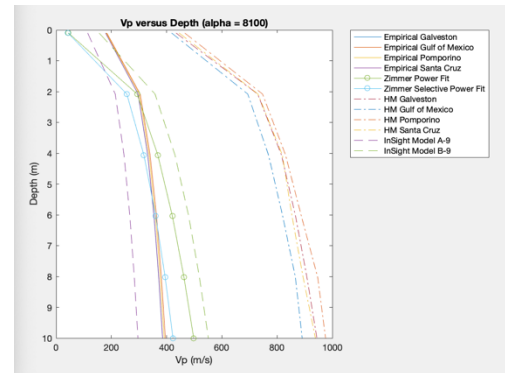


Figure 3: Same visual scheme as Fig. 1. The empirical correction is now calculated using $\alpha = 8100$. This results in a profile with more velocity overlap, but less geometrical overlap.

From this work, we conclude that at shallow depths, unconsolidated sediments on Mars are quite similar to those on Earth. Velocities measured in the lab (11) of spherical sand grains show velocity magnitudes that are within the bounds of possible velocity profiles at the InSight site (12). After applying a simple empirical correction to account for the Hertzian overprediction, one can recreate the structure and magnitude of the velocity profile for both a martian and terrestrial case. This method holds the most promise for further modelling which would introduce the presence of cements such as water ice.

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