AN ADVANCED REFLECTIVITY-BASED PERMITTIVITY ESTIMATION FOR PLANETARY RADAR

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Introduction: Ground-penetrating radar (GPR) is a mainstream scientific payload in both Lunar and Martian missions, with 3 Martian satellite radars (Tianwen-1, MARSIS, SHARAD) and currently 3 active in-situ radar units (RIMFAX, Yutu-2, Zhurong) [1]. GPR aims at mapping the dielectric properties of the investigated medium in an indirect manner. The dielectric properties of a linear, isotropic and non-dispersive medium can be sufficiently described by its electric permittivity (ϵ) and conductivity (σ) [2]. The electric permittivity ϵ is associated with the electromagnetic velocity and the reflection coefficient, and is also directly related to the density and the minerological content of the medium [1].

The most common approach for estimating the permittivity using common-offset (CO) GPR is Hyperbola Fitting (HF) [3]. Despite its popularity, HF has many drawbacks and limitations [3]. The most important of which is the fact that in order for HF to work it needs distinct point-like targets. These are often absent, or their signatures are masked with noise and clutter. To that extent, the applicability of reflectivity-based permittivity estimation was recently explored in the literature [4]. This is mature technique that was initially developed and tuned for applications in highway engineering [2]. In the current paper, via a numerical case study, we explore the accuracy and the limitations of the conventional reflectivity-based approach, and we demonstrate that it cannot be reliably applied in planetary radar. Subsequently, we suggest an advanced set of methods that overcome these limitations and improve upon the accuracy and the applicability of the conventional approach.

Conventional Approach: The reflection coefficient of a plane-wave over a homogenous half-space equals with $R = \frac{\sqrt{\epsilon_1}-\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}+\sqrt{\epsilon_2}}$, where ϵ_1 is the permittivity of the medium carrying the incident and the reflected field, and ϵ_2 is the permittivity of the second layer. If the first layer is free-space ($\epsilon_1 = 1$), is easy to prove that

$$\epsilon_2 = \left(\frac{1+R}{1-R}\right)^2 \quad (1)$$

The reflection coefficient can also be calculated by $R = -{A_r}/{A_{pec}}$, where A_r is the amplitude of the reflected field from a homogenous half-space with $\epsilon = \epsilon_2$ at distance h, and A_{pec} is the amplitude of the reflected field over a metallic plate at distance h. Conventional reflectivity-based approach, estimates A_{pec} via a calibration measurement over a metallic plate, and then estimates the permittivity of the investigated medium using the amplitude of the reflection A_r. This approach is based on two assumptions A) that the incident field is a plane-wave, and B) that the amplitudes of A_{pec} and A_r are not corrupted with cross-coupling or other reflections from the surrounding targets. Via a numerical case study, we will explore how these assumptions affect the performance of reflectivity-based permittivity estimation, and we will suggest alternative approaches to mitigate these effects.

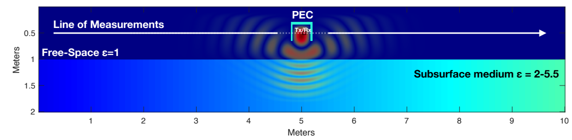

Figure 1: The investigated numerical case. An antenna with 500 MHz central frequency is placed at 0.5 meters above the ground. The investigated medium is a half-space with linearly varying permittivity $\epsilon = 2 - 5.5$ from left to right.

Numerical Case Study: The investigated numerical case study is shown in Figure 1. The antenna has 500 MHz central frequency and is placed at 0.5 meters above the ground. The investigated medium is a half-space with linearly varying permittivity $\epsilon = 2 - 5.5$ from left to right. The antenna is shielded without any absorbers in order to simulate the repetitive reflections and unwanted clutter in rover-coupled antennas. The simulations were executed with and without the shielding in order to investigate the effects of different types of clutter to the estimated permittivity.

Figure 2 shows the real and the estimated permittivities using the conventional reflectivity-based method. For both cases, A_{pec} was calculated by simulating a perfect electric conductor (PEC), and both A_{pec} and A_r were calculated from the highest peak of the reflected signal. The results, for both cases, greatly deviate from the ground truth, indicating that the conventional reflectivity-based approach as applied in highway engineering is not directly applicable in planetary radar. A potential source of error in Figure 2 is the violation of the main assumptions discussed in the previous section i.e. A) plane wave source, and B) that the measured A_{pec} and A_r are not affected by the cross-coupling and any external reflectors. It will be shown in the next examples that the plane-wave assumption does not affect the accuracy of the results. The main source of error occurs because instead of calculating $R = -A_r/A_{pec}$ we

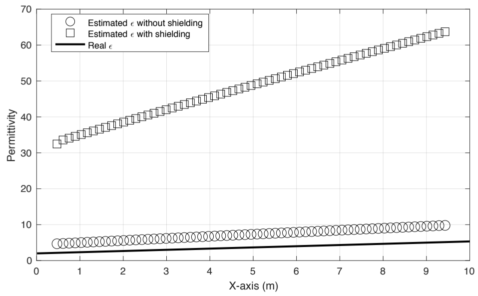

Figure 2: The estimated permittivities for with and without PEC shielding for the numerical case study shown in Figure 1. The permittivities are estimated using the conventional reflectivity-based approach.

calculate $-\frac{M_r}{M_{pec}} = -\frac{A_r+CP}{A_{pec}+CP}$, where CP is the cross-coupling plus any external reflectors. In the following paragraphs we suggest 3 novel methods for estimating CP, which is subsequently subtracted from the signals M_r and M_{pec} in order to approximate the reflection coefficient $R = -\frac{A_r}{A_{pec}}$. Finaly, R is used in (1) to estimate the bulk permittivity (ϵ_2) of the investigated half-space.

Method A: In the first approach we approximate CP with the free-space response of the antenna. Instead of just taking one calibration measurement over a metallic plate (M_{pec}), we now need an additional measurement in free space ($\approx CP$), which is subsequently subtracted from both M_r and M_{pec} before calculating R. The results are shown in Figure 3. Both with and without the PEC shielding, the results are in excellent agreement with the ground truth indicating the reliability of this approach. The main drawback of this method is that it requires an additional calibration measurement in free space, something that is not always plausible in planetary radar due to practical constrains.

Method B: The second method does not require the additional free-space calibration. Instead, it utilises $M_{c1} = A_{c1} + CP$, where M_{c1} is the measurement over a medium with known permittivity, and A_{c1} is the pure reflection from this surface without cross-coupling and external reflections. M_{c1} can be measured during the mission over a medium with clear and visible hyperbolas, where HF can be used to estimate the permittivity of the area.

The measured signal over the metallic plate equals with $M_{pec} = A_{pec} + CP$. At the area with the known permittivity, the measured field equals with $M_{c1} = A_{c1} + CP$. Moreover, based on the reflection coefficient formula ${}^{A_{c1}}/_{A_{pec}} = -\frac{1-\sqrt{\epsilon_{c1}}}{1+\sqrt{\epsilon_{c1}}}$, where ϵ_{c1} is the known permittivity. From the above, a system of three equations and three unknowns is derived

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & \frac{1-\sqrt{\epsilon_{c1}}}{1+\sqrt{\epsilon_{c1}}} & 0 \end{bmatrix} \begin{bmatrix} A_{c1} \\ A_{pec} \\ CP \end{bmatrix} = \begin{bmatrix} M_{c1} \\ M_{pec} \\ 0 \end{bmatrix} \qquad (2)$$

CP and A_{pec} can be calculated from solving (1), and CP can be subsequently subtracted from M_r to approximate A_r. Figure 3 shows the results assuming that the permittivity at $x = 1$ m (see Figure 1) is known and equals with $\epsilon_{c1} = 2.2$. The estimated permittivity throughout the scan is in excellent agreement with the ground truth, without the need for an additional free-space calibration.

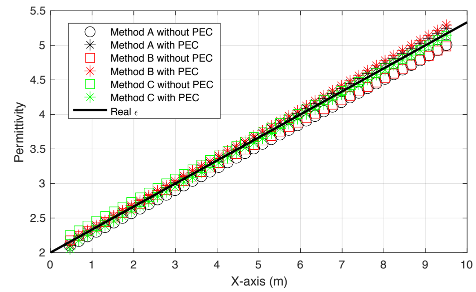

Figure 3: The estimated permittivities of the numerical case study shown in Figure 1. The results are shown for both with and without PEC shielding using the novel *Methods A, B, C* as described in the text.

Method C: The last method removes the need for M_{pec}, since a re-callibration over a metallic plate is often unavailable for on-going missions, and for many missions the calibration response is not available to the public. In this method we use the same rationale as in *Method B* but instead of one area with known permittivity (using HF), we use two. This leads to the following system of equations:

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & \frac{1-\sqrt{\epsilon_{c1}}}{1+\sqrt{\epsilon_{c1}}} \\ 0 & 1 & 0 & \frac{1-\sqrt{\epsilon_{c2}}}{1+\sqrt{\epsilon_{c2}}} \end{bmatrix} \begin{bmatrix} A_{c1} \\ A_{c2} \\ CP \\ A_{pec} \end{bmatrix} = \begin{bmatrix} M_{c1} \\ M_{c2} \\ 0 \\ 0 \end{bmatrix} \qquad (3)$$

where M_{c1} and M_{c2} are the measurements over the two areas with known permittivity; and ϵ_{c1} and ϵ_{c2} are the permittivities of the two areas. The system in (4) is a well-determined and well-posed system of equations that can be solved to approximate CP and A_{pec}. CP is subsequently subtracted from the measurements to get A_r. Notice that A_{pec} is indirectly evaluated from (1) without the need for a calibration measurement over a metallic plate. The results using this approach are shown in Figure 3. We assume that the permittivity at $x = 1$ m and $x = 9.5$ meters are known i.e. $\epsilon_{c1} = 2.2$ and $\epsilon_{c2} = 5.3$. The results throughout the scan are in excellent agreement with the ground truth without the need for any calibration measurement. This makes the suggested method very appealing due to the fact that antenna units need to be re-calibrated after long periods of usage. This poses no problem for earth applications, but re-calibration over metallic plates is unattainable in currently active planetary missions.

References [1] I. Giannakis, 2021, *GRL*, [2] D. Daniels, 2005, *Springer*, [3] I. Giannakis, 2021, *IEEE GRSL*, [4] T. Casademont, 2022, *AGU*.