TESTING PERMEABILITY-POROSITY RELATIONSHIPS FOR MODELING ICY SHELLS.
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Motivation: Ocean worlds are prevalent in the outer Solar System. Studies of Europa and Enceladus both observational and theoretical show that subsurface oceans are likely salty. The growth of an ice shell from a saline ocean occurs in dendritic fashion, like branches of a tree, leading to the formation of a two-phase "mushy zone" at the ice/ocean interface [1]. Properties of this mushy zone, particularly porosity and its relationship to permeability, are important for the transport of solutes during freezing, which in turn affects the thermal, chemical, and geophysical properties of the forming ice shell. Therefore, to accurately model an ocean world's ice shell, it is important to understand the relationship between porosity and permeability.

However, the functional form of this relationship is not well constrained for porous icy systems. Moreover, these properties are not uniform across an ice shell, and small-scale heterogeneities can lead to the formation of high porosity brine channels that efficiently move high-salinity brine through the mushy zone. Any model looking to accurately capture the properties of an ice shell must be able to recreate these small-scale features and appropriately parameterize their effects on salt transport at the ice-ocean interface.

Most models of the icy shells of Europa and Enceladus have concentrated on pure ice, ignoring the contribution of salts known to be present. Modeling icy shells as a multiphase system is a step forward in understanding the growth and composition of these complex systems. The relative inaccessibility of the outer Solar System means many researchers rely on predictive model data to help interpret existing spacecraft observations and inform future investigations. Understanding the limits of the models we use and the effect of different functions and parameters is an important part of verifying their reliability and suitability for use by the planetary science community.

Methods: Permeability (K) is generally expressed as a function of porosity (χ), and several relationships between porosity and permeability have been suggested. In this study, we examine three different porosity-permeability relationships using a reactive porous media model to see their impact on the results of modeling the growth of a salt-rich ice shell. We use the Solidification, Flow, and Thermodynamics in Binary Alloys (SOFTBALL) model to simulate phase evolution, heat transport, and mass transport in a growing ice shell. [2] Earth ocean parameters for salinity and temperature were used in order to benchmark results against observed sea ice behavior. Three porosity-permeability functions are built into SOFTBALL: Kozeny-Carman, cubic, and logarithmic. [2,3] The general behavior of each function is shown in Figure 1.

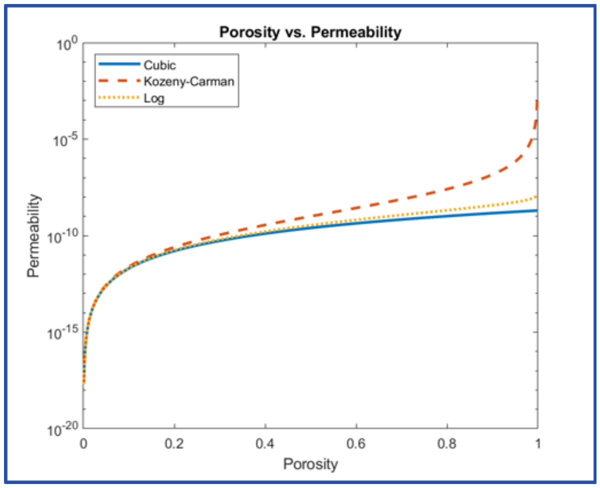

Figure 1: Porosity vs permeability plotted for the three functions examined in this study.

SOFTBALL simulates flow in Hele-Shaw cells to create a quasi-2D flow model of convection in a binary alloy, where sufficiently narrow cell widths create flow patterns that obey Darcy's law. Because the Hele-Shaw cell width (d) is an input variable for the model, we tested each of the three porosity-permeability functions at different cell widths to determine the impact of cell width on the results of the functions. The model simulates top-down ice growth over time and outputs 2D spatial maps of porosity and bulk salinity at each timestep (e.g., Figure 2). We compare each porosity-permeability function's results for ice layer thickness, mushy zone thickness, mushy zone fractional thickness (mushy zone thickness/total ice shell thickness), and bulk salinity over time. The number and spacing of brine channels were also calculated, brine channels being inferred based on spikes in porosity values approaching χ=1. Figure 2 shows a sample spatial map for porosity.

Initial runs were completed using sea ice parameters validated by [4] with d=5x10-5 m for a 2 m square domain. Additional model runs were then conducted for each porosity-permeability function with the same parameters but varying d values: from d=3.5x10-5 m to d=2.0x10-4 m, with additional runs at intermediate

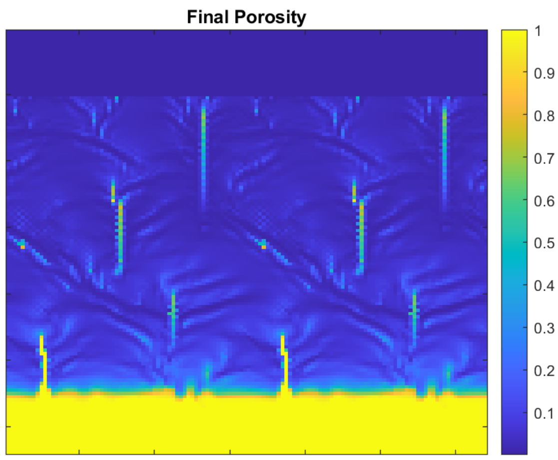

Figure 2: A sample spatial map of final porosity, showing solid ice at the top (porosity ~0), liquid water at the bottom (porosity ~1), and the mushy zone in the middle with varying porosity and brine channels.

values to verify continuous behavior.

Results: *Results at low cell spacings.* At low Hele-Shaw cell spacings (up to ~10^{-4} m), results for all three porosity-permeability functions were nearly identical. Ice layer thickness, mushy zone thickness, and mushy zone fractional thickness were all equal. Bulk salinity values varied within the mushy zone by a few ppt, which is within the variation between multiple runs with a single function. All three functions resulted in similar numbers of brine channels formed with consistent spacing over the domain. At early times many brine channels are formed very close together, and as time progresses the number of brine channels decreases, eventually converging to a steady state of a few evenly spaced channels (6-8 channels for our 2 m domain).

Results at increased cell spacings. As Hele-Shaw cell spacing increased to $2\text{x}10^{-4}$ m, we found a slight divergence in the results of the cubic function compared to the Kozeny-Carman and logarithmic functions (Figure 3). In the cubic function, the ice grows more quickly and overall shell thickness is increased by a small amount. However, results for mushy zone fractional thickness and bulk salinity remain consistent between all functions. Increased cell spacing has the biggest effect on brine channel formation. None of the three functions show the pattern of many brine channels at first evolving into a steady number over time. Brine channel numbers and spacings are highly variable, and the Kozeny-Carman and logarithmic functions both show delayed brine channel formation compared to the previous results.

Results at largest spacings. At Hele-Shaw cell spacings of 1.5 mm and above, fluid flow is less restricted, resulting in high fluid velocities in the mushy zone. At this point, computation time for model runs increases drastically (from hours to several weeks) and model results become unstable or unphysical. Changes in other parameters should be investigated to determine whether failure at this cell width is a limit of the model or of the Hele-Shaw method itself.

Conclusion: Model validation has shown that the three permeability functions tested are consistent when used in a multiphase model that utilizes Hele-Shaw flow, however as cell spacing increases the functions begin to diverge. As cell spacings approach the cm scale, Hele-Shaw flow may no longer be applicable to modeling icy shells. These tolerance levels are important to understand as predictive models are necessary for planning outer Solar System missions such as the Uranus Orbiter and Probe flagship.

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References: [1] Feltham, D.L. (2006) *GRL* 33.14. [2] Parkinson, J. (2019) *Oxford University*. [3] Katz, R.F., Worster, M.G. (2008) *Journal Comp. Phys.* 227, 9823-9840. [4] Buffo, J. et al (2021) *JGR Planets, 126*.

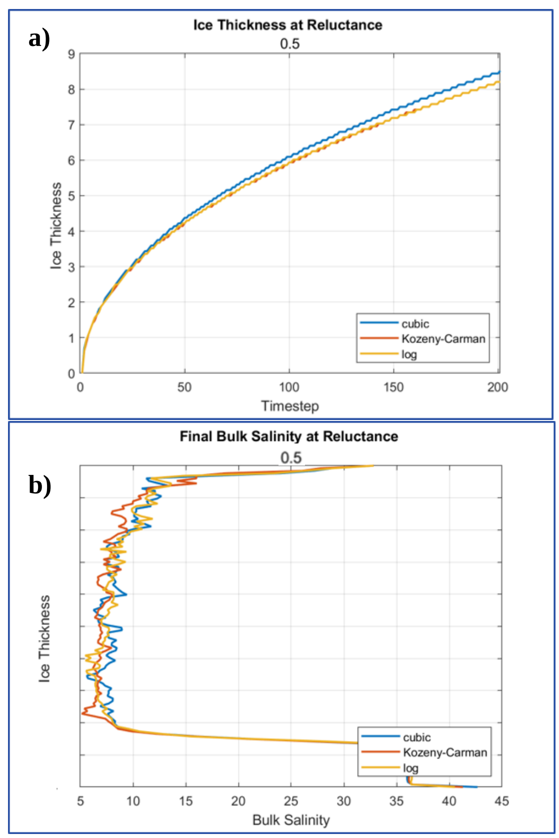

Figure 3: a) slight divergence of the cubic function for ice thickness at d=0.2e-4 m. b) Final bulk salinity (ppt) of the ice shell; surface is at the top. Reluctance is proportional to 1/(d^2). Units are non-dimensional.