Investigating groundwater dynamics and residence times on early Mars using unconfined aquifer model with vertical heterogeneity. M. A. Shadab[1,2,3,†], E. Hiatt[2,3,4,*], and M. A. Hesse[1,2,4,‡], [1]Oden Institute for Computational Engineering and Sciences, [2]Institute for Geophysics, [3]Center for Planetary Systems Habitability, [4]Department of Geological Studies, Jackson School of Geosciences, The University of Texas at Austin, Austin TX ([†]mashadab@utexas.edu, [*]eric.hiatt@utexas.edu, [‡]mhesse@jsg.utexas.edu).

Introduction: Earlier we developed an analytic solution for a homogenous, steady unconfined groundwater aquifer beneath the southern highlands of Mars [1] and used it to explore self-consistent combinations of mean recharge and mean hydraulic conductivities. Our results showed that due to spherical geometry even a comparably small amount of recharge raises the water table to the mean land surface for all suggested shorelines of a hypothesized ocean in the northern lowlands. These solutions have been validated with extensive, quasi-3D simulations with complex shorelines and craters [2].

Here we extend our model to study a crustal aquifer with a vertical heterogeneity [3] as a power-law decay in porosity and permeability with depth. Adding this complexity greatly affects the groundwater dynamics inside the aquifer. In addition to solutions for global groundwater table height, we derive the analytic solutions for the residence times and streamlines of the flow inside the aquifer. Our results show that a decay in permeability with depth skews the dynamics to near-surface, increases the height of the groundwater table thus making it more susceptible to a widespread seepage and substantially decreases the residence time.

Aquifer model: We assume a crust with vertical variation in porosity and permeability with depth as $\phi = \phi_0 z^m$ and $k = k_0 z^n$ respectively. Here z is the elevation from the impermeable base and ϕ_0, k_0, m and n are model constants. The Mars crustal data presented in [3] gives $m = 2.54$ and $n/m = 3$ and base values $\phi_0 = 3.39 \times 10^{-11}$ with reference permeability of $k = 10^{-12.65}$ m^2 at 1 km depth.

Similar to other Mars groundwater models, we use the Dupuit-Boussinesq model [4-6] with power-law decay [7] for the elevation, h, of the groundwater table above the base of the aquifer

$$\frac{\phi_0}{m+1}\frac{\partial h^{m+1}}{\partial t} - \nabla \cdot \left(\frac{K_0}{n+1} h^{n+1} \nabla h\right) = r. \qquad (1)$$

Here, K_0 is the hydraulic conductivity of the aquifer at unit elevation, given by $\frac{k_0 \rho g}{\mu}$ with ρ and μ being density and viscosity of liquid water and g being the mean acceleration due to gravity on Mars. Moreover, r is the mean recharge over the entire highlands. The divergence and gradient operators take the standard form in spherical shell coordinates. We assume the flow is azimuthally symmetric so that the solution is only a function of the southern co-latitude, θ.

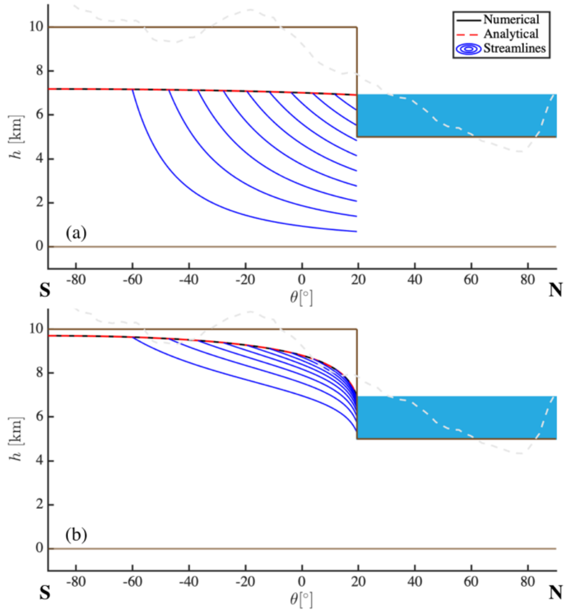

Figure 1: *Analytic solutions for steady unconfined aquifer on spherical shell with* $r = 4 \times 10^{-6}$ *m/Earth year and* $R = 3{,}389.5$ *km for Arabia shoreline, i.e.,* $\theta_o = 110°$ *(*$20°$ *N) and* $h_o = 6910$ *m. a) Homogenous case and (b) Heterogeneous case with vertical variation. The azimuthally averaged topography from MOLA (gray dashed line) and the assumed step profile with impermeable base (brown) are also shown.*

The boundary conditions are no-flow due to symmetry at the south pole, $\theta = 0$, and the elevation of the presumed ocean, h_o, in the northern lowlands at the shoreline, θ_o. We assume a crustal aquifer with a depth of 10 km and a base at 0 km elevation. Further, we assume a simple step function topography with elevations of 10 km and 5 km in the southern highlands and northern lowlands, respectively (Fig. 1a). The change in topography occurs at, $\theta_o = 110°$ ($20°$ N), the average southern co-latitude of the Arabia shoreline. Here, the groundwater is in contact with the hypothesized ocean shown in light blue color. First, we explore the effect vertical heterogeneity on the height of water table, then the groundwater dynamics from stream lines and lastly the residence times of water parcels.

Results: The analytic solution presented below is a unique opportunity to interrogate the effect of vertical heterogeneity on the height of water table. Moreover, we will implement this solution to further investigate the dynamics of groundwater inside the aquifer.

Analytic solution. We explore the steady solution of the Dupuit-Boussinesq equation on a spherical cap with radius R. The elevation, h, of the groundwater table above the base of the aquifer is

$$h(\theta) = \sqrt[n+2]{h_o^{n+2} + (n+2)\, h_c \, ln \left| \frac{\sin\theta \, \tan(\theta_o/2)}{\sin\theta_o \, \tan(\theta/2)} \right|} \quad (2)$$

with the characteristic height, $h_c = \left(\frac{rR^2(n+1)}{K_0}\right)^{\frac{n+2}{n+1}}$.

Solutions for the groundwater table without and with vertical heterogeneity are shown in Figure 1a and 1b respectively with black (numerical) and red (analytical) lines. The analytical results show an excellent comparison with numerical solutions of Equation (1).

For a same recharge rate, the crust with vertical decay in porosity and permeability with depth significantly increases the height of the groundwater table in comparison with the homogeneous case (Figure 1). It happens because the decay in permeability drops the mean hydraulic conductivity of the crust leading to a build-up of groundwater. This rise in the groundwater table reduces the plausible groundwater recharge rates avoid widespread groundwater upwelling compared to the homogenous case.

Effect of vertical heterogeneity on fluid dynamics

From the height of the groundwater table (Eqn. 2) and Darcy's law, the streamfunction can be derived as

$$\psi(\theta, z) = \frac{K_0}{n+1} h_c^{n+2} \left(\frac{z}{h}\right)^{n+1} (\cos\theta - 1). \quad (3)$$

The streamfunction, $\psi(\theta_s, z_s)$=constant, gives the locus of the streamline emanating from the top of water table at θ_* with its height, z_s, and location, θ_s related as

$$z_s(\theta_s) = h(\theta_s) \sqrt[n+1]{\frac{1-\cos\theta_*}{1-\cos\theta_s}}. \quad (4)$$

The streamlines show how water entering the water table at θ_* seeps to the Northern lowlands on the right (Figure 1, blue lines). Streamlines are plotted so that the streamtube between any two streamlines carries the same volume of groundwater. For the homogeneous aquifer (Figure 1a), the streamlines are widely and almost uniformly spaced indicating that an equal amount of water is being carried and most of the pore space is accessible.

For the case of vertical heterogeneity, the streamlines have come much closer near the surface, indicating a higher flux shallow path. This shows that the deeper crust is inaccessible for the groundwater due to reduced pore space and permeability. Thus, most groundwater dynamics occurs near the surface.

Residence times of groundwater inside the aquifer

Using streamline locus and Darcy's law, the residence time, t_r, can be derived by integration along the streamline as

$$t_r(\theta^*) = \frac{\phi_0}{K_0} \frac{R^2}{h_c^{n+2}} \int_{\theta_*}^{\theta_o} \frac{h(\theta_s)^{n+1} \sin\theta_s}{z_s(\theta_s)^{n-m}\,(1-\cos\theta_s)} \, d\theta_s. \quad (5)$$

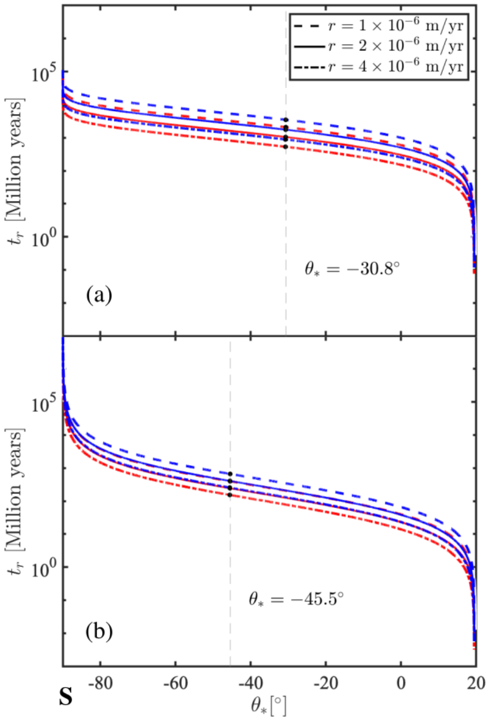

Figure 2: *Semi-analytical solutions for residence times of water entering the aquifer at θ_* for a) homogenous and (b) heterogeneous cases. The red and blue colored lines correspond to the surface porosities, $\phi(z = 10$ km), of 0.3 and 0.5 respectively. The gray line shows the location of volumetric-averaged residence times.*

The average residence times have dropped by almost an order of magnitude from 1-10 Ga (Figure 2a) to 0.1-1 Ga (Figure 2b) due to presence of vertical heterogeneity. The parcel entering at the pole resides for the maximum time ($\theta_* = -90°$ N). Moreover, increasing the recharge rate leads to a faster drainage.

Discussion: Our results show that the vertical heterogeneity leads to a higher water table than that for the crust with homogeneous properties. This leads to a reduction in the critical recharge rates provided in the published estimates [1,2]. Furthermore, the vertical heterogeneity substantially increases the residence times and restrict the fluid dynamics to near-surface.

References: [1] Shadab et al. (2022) LPSC 2022, Abs #1775. [2] Hiatt et al. (2022), Authorea Preprints. [3] Manning and Ingebritsen (1999), Revs of Geophysics, 37(1), 127-150. [4] Clifford (1993) JGR, 98(E6), 10973- 11016. [5] Hanna et al. (2005) JGR, 110(E1). [6] Luo and Howard (2008) JGR, 113(E5). [7] Zheng et al. (2013), J. Fluid Mech., 718, 558-568. [8] Carr and Head (2003) JGR, 108(E5).