SCALING RELATIONSHIPS FOR MIXED HEATED MANTLE CONVECTION WITH APPLICATION TO THE EVOLUTION OF TERRESTRIAL PLANETS.

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Introduction: The thermal, tectonic, and magmatic history of a terrestrial planet is connected to thermal convection within its rocky mantle layer. Mantle convection can be driven by internal and/or basal heating. That is, heat generated internally by the decay of radiogenic elements (tidal heating can also act as, effectively, an internal heat source) and heat flowing into the base of the mantle from a planet's core. Mixed heating occurs when both basal and internal heating are present.

Investigations into the dynamics of thermal convection using a combination of boundary layer theory and numerical experiments can be used to develop scaling relationships and parameterizations that can be used to model the thermal evolution of terrestrial planets and moons [1]. The parameterizations are often based on the end-member limit of pure basal or pure internal heating under the assumption that they will adequately capture the dynamics of mixed heating. An aspect of that assumption is the idea that boundary layer dynamics are self-determined [5].

Moore [2] provided theoretical scaling relationships for mixed heating convection, combined with numerical convection experiments in 2D cartesian domain, that challenged the assumption of self-determined boundary layers. Boundary layer dynamics in the Moore [2] study were influenced by boundary layer interactions that extended to degrees of convective vigor expected within terrestrial planets akin to the Earth. Lenardic et al. [3] extended the Moore scaling relationships to include surface velocities. The scalings, tested against numerical experiments in 3D cartesian modeling domains, showed that boundary layer interactions in a mixed heating system could lead to velocity scaling trends distinctly different from that of end-member heating modes. This study expands the theory found in Moore [2] and Lenardic et al. [3] and its testing via numerical convection experiments in 3D spherical modeling domains. Our main objective is to use the combination of theory and numerical calculations of mixed-heated convection to derive calibrated scaling relationships for the average internal temperature, basal and surface boundary layer temperature, basal and surface heat flux, and surface and root mean square (rms) velocities as a function of internal heating (H) and Rayleigh number (Ra). We will also explore how changes in convective planform and wavelength can alter velocity scaling relationship. We will end with thoughts on how our results can affect modeling the thermal-magmatic evolution of terrestrial planets.

Methods: Theoretical scaling relationships were calibrated to numerical data to obtain values for the free scaling coefficients that depend on system geometry (i.e. Spherical vs Cartesian). To model convection in 3D spherical coordinates, we employed the CitcomS 3.2.0 finite element code published under the GPL2 license [4]. The model domain consists of a spherical shell that represents the mantle and lithosphere. Dimensions are set to the mantle thickness of the Earth. The full mesh is formed by 12 spherical caps; each cap is formed by n x n x n grid elements. The model is defined by the non-dimensional equations of conservation of mass,

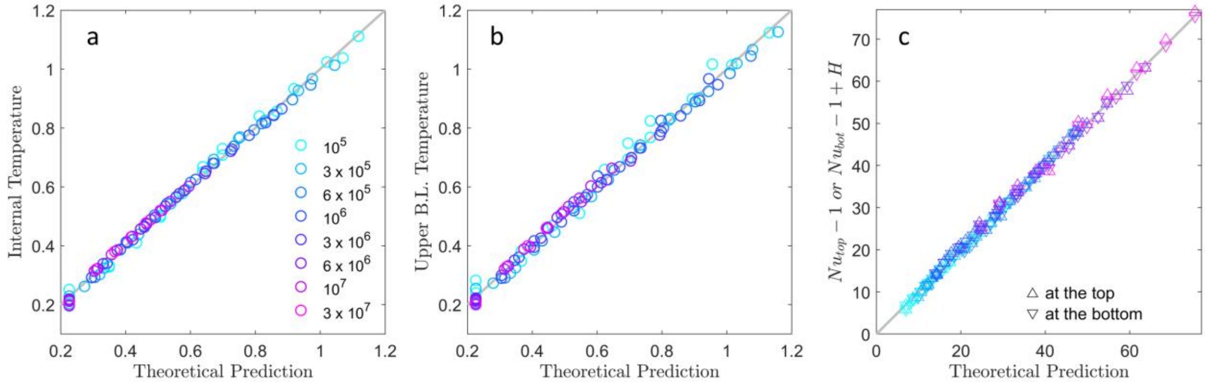

Figure 1. Shows the numerical simulation results in the y-axis, and the values predicted by the theory in Moore (2008) in the x-axis for internal temperature (a), upper boundary layer temperature (b), and Nusselt number (c). The grey line denotes a 1:1 correspondence.

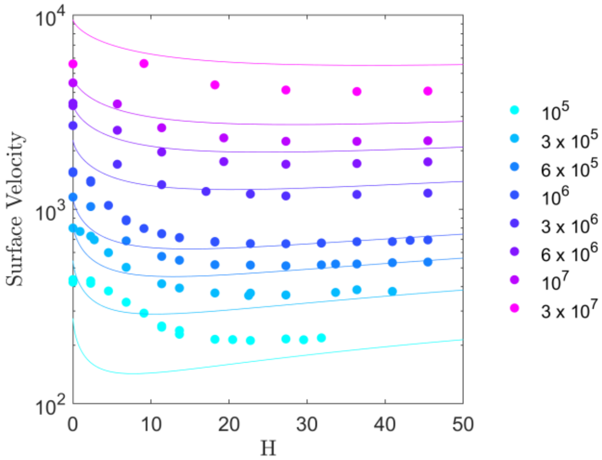

Figure 2. Shows the model output data at statistical steady state for surface velocity and the theoretical predictions as a function of H

momentum, and energy for mixed heating incompressible mantle convection. For all numerical experiments the surface and basal temperatures were kept constant at non-dimensional values of 1 and 0 respectively. The boundary conditions are free-slip, and the viscosity was kept the same throughout.

Results: The results for temperature and heat flux match closely with the 2D, cartesian results in Moore [2] with small differences due to the change in geometry (Figure 1). Results for velocity show that it decreases as H is increased, which contradicts classical convection theory [5]. For high Ra cases, the scaling relationship from Lenardic et al. [3] matches well with the numerical results from this study (Figure 2). However, for low Ra cases and low H values, our results begin to quantitatively differ from scaling theory with surface velocity increasing more steeply than that predicted by Lenardic et al. [3]. Changes in planform and wavelength, with convective vigor lead to these differences [6]. More specifically, changes in the morphology of mantle downflows, from sheet-like to plume-like, lead to significant changes in surface velocities – a factor that has not been included in theoretical scalings to date.

A key result is that decreasing surface velocity with increased internal heating within the mantle is shown to be robust and independent of system geometry. This runs counter to traditional scaling parameterizations used to the model the thermal histories of terrestrial planets and moons. The robustness of that result also indicates that boundary layer interactions can affect mantle convection dynamics at degrees of convective vigor similar to present day Earth. That potentiality is also not accounted for in the majority of thermal history parameterizations.

Conclusion: This work tested and validated the theory found in Moore [2] and Lenardic et al. [3] for mixed heating isoviscous convection on a 3D spherical domain. The results for temperature and heat flux compare well with previous data. The scaling relationship for the surface velocity works well for high H and Ra, but have larger discrepancies at lower H and Ra values.

Our results show that the trend of surface velocity decreasing with increased internal heating is robust for convection in a sphere and that interaction between boundary layers (i.e., the lithosphere and mantle plumes) can affect the dynamics of mantle convection at high degrees of convective vigor. Both these results run counter to assumptions often used in modeling the thermal and magmatic histories of terrestrial planets.

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