DART IMPACT: SETTING CONSTRAINTS USING THE TSIOLKOVSKY ROCKET EQUATION.

K.R. Ramsley[1,2] and J.W. Head[1]. [1]Department of Earth, Environmental and Planetary Sciences, Brown University, Providence, RI 02912 USA. [2]School of Engineering, Brown University, Providence, RI 02912 USA, Kenneth_Ramsley@brown.edu.

Introduction: The Tsiolkovsky Rocket Equation applies Newton's Second Law of Motion to the study of an accelerating body where a change in the velocity of the body (delta-v) is excited by the ejection of mass from the body. An accelerating impulse that depletes body mass may arise from an energetic reaction within a body, such as a rocket engine expelling combustion products, or from an external energy source such as an impacting projectile that launches ejecta from a crater. The Rocket Equation includes three factors: 1) the pre-event mass of the body, 2) the average velocity of expelled mass, and 3) the body's post-acceleration-event mass. [1]

We may apply the Rocket Equation to infer a missing term. For example, if we know the body's pre-event mass, the rocket nozzle's average velocity, and the body's post-acceleration delta-v, we can calculate the unknown mass of accelerating propellant. Similarly, at the conclusion of an accelerating event, if we know the input momentum and momentum transfer efficiency of the rocket engine, we can calculate the body's pre-event mass.

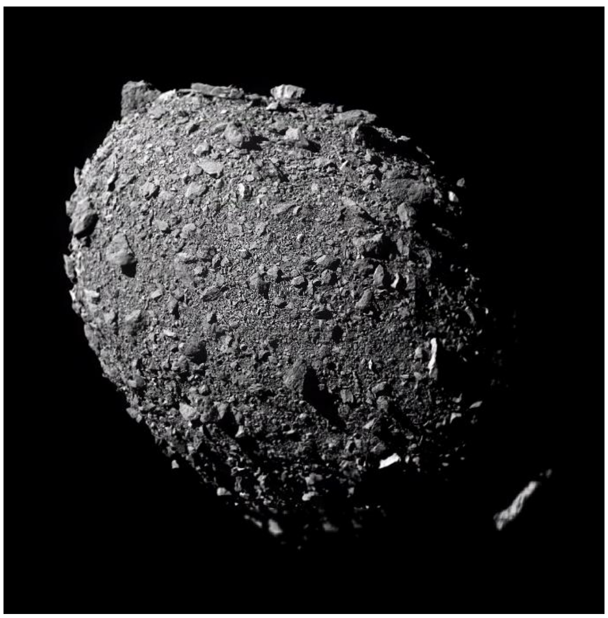

Fig. 1: Dimorphos (D ~170 m), satellite of Asteroid 65803 Didymos (D ~570 m). Composite image from Demonstration of Autonomous Rendezvous Technology (DART) spacecraft images seconds prior to impact. *Johns Hopkins University Applied Physics Laboratory*.

Pre-impact parameters of the mass and velocity of the Demonstration of Autonomous Rendezvous Technology (DART) spacecraft are closely constrained [2]. Consequentially, where the post-impact delta-v of the Dimorphos target body is also closely constrained via ground-based observations [3], we predict the mass of Dimorphos using the Rocket Equation.

Where images of the DART impact crater from ESA's Hera spacecraft [4] are likely to suggest the mass of missing crater ejecta, the observed crater will set limits on the average velocity of the excited ejecta [5].

The Rocket Equation does not predict the size of the DART impact crater on Dimorphos. However, where ~60% of the DART impact momentum likely excited Dimorphos' delta-v [6], impact crater ejecta momentum represents approximately 60% of the spacecraft's momentum – and this sets a limited range of possible missing ejecta mass.

Where momentum is conserved, the observed volume of the DART crater will suggest the average velocity of ejecta. For example, assuming the pre-impact-predicted density of Dimorphos, if average ejecta velocity were 50 m/s, missing ejecta mass would produce a D ~10-15 m crater. If ejecta velocity were 10 m/s, then a D ~20-25 m crater, and if 3 m/s, then a D ~30-40 m crater [5].

Observations from the Hera mission are likely to closely constrain the post-impact mass properties of Dimorphos (mass, volume, and consequently – density). Taken together (Dimorphos mass properties, DART impact momentum, DART crater volume, ejecta mass and velocity) – this will set narrow limits on the physical properties of Dimorphos in particular, and inform crater formation models on low-density bodies in general [5].

Method: Our application of the Tsiolkovsky Rocket Equation is as follows…

$$\triangle v = v_{exh} \ln(\frac{M_0}{M_1})$$

A rocket's change in velocity (Δv) equals (V_{exh}) times the natural log of M_0 / M_1 …where V_{exh} is the velocity of the engine's exhaust, M_0 is the starting mass of the rocket including propellant mass, and M_1 is the rocket's mass minus the expended propellant mass.

In our study:

Rocket = The Target Body (Dimorphos).
Propellant = Projectile Mass (DART Spacecraft)
…*or* Crater Ejecta Mass (crater not yet observed).
Exhaust Velocity = Spacecraft Impact Velocity
…*or* Average Ejecta Launch Velocity (not observed).

We begin with the DART spacecraft mass and the Dimorphos intersection velocity. From this input, plus the observed volume of Dimorphos and its post-impact delta-v – combined with a 60% 'rocket engine' efficiency [6], we calculate the mass of Dimorphos. Based on these inputs and results, we predict the average bulk density of Dimorphos and its percentage of interior porosity.

The momentum of the DART spacecraft is the source of the propulsive force. However, the delta-v of Dimorphos is most directly produced by mobilized impact ejecta. The accelerating force provided by the ejecta includes vaporized, spalled, and excavated material [5] minus losses that result in a less efficient transfer of momentum. These losses include [6]:

-Cone-shaped ejecta launch angles (the largest loss).
-Horizontal ejecta jetting.
-Target compression and displacement.
-Heating, melting, and vaporization of target material.
-Rock fracturing and mineral shock metamorphosis.
-Oblique impact angle and/or sloping target terrain.
-Spallation of material from the target's opposite hemisphere.

Results and Discussion: The DART spacecraft intersected a central location of Dimorphos on a surface with no sloping terrain [2]. Further, Dimorphos appears to be a symmetrical body [Fig. 1], which suggests an impact site and vector angle that minimized the inefficiencies of an oblique impact.

Pre-impact studies of Dimorphos predict a spherical equivalent diameter of 0.171 ± 0.011 km [7]. Assuming a bulk density of ~2.17 g cm^3 (assumed to be the same as the density of Didymos) [8], pre-impact predictions suggest a Dimorphos mass of ~5.68 x 10^9 kg [9].

The DART spacecraft intersected Dimorphos with a mass of 570 kg and velocity of 6,258 m/s [2]. Based on models of the Dimorphos' pre-impact orbit [2], pre-impact predictions of the asteroid's mass, the known momentum of the DART spacecraft, and a likely momentum transfer efficiency of 60%, our application of the Tsiolkovsky Rocket Equation agrees with a consensus of pre-event predictions – a Dimorphos orbital period reduction of ~200 seconds, which corresponds to a post-impact delta-v of ~0.5 mm/s [2].

Post-impact observations *substantially* diverge from the predicted outcome. Instead of a Dimorphos orbital period reduction of ~200 seconds, the observed post-impact orbital period of Dimorphos is reduced by **1,930** seconds ± 120 sec [2], and instead of a predicted delta-v of ~0.5mm/s, the delta-v is **~2.5** mm/s.

Where the momentum of the DART impact is closely constrained by the mass and velocity of the spacecraft and the volume of Dimorphos is based on reasonably accurate observations – it is very likely that the average density of Dimorphos is **5X less** than pre-impact predictions. In fact, even if the volume of Dimorphos is on the extreme low end of pre-impact predictions, its density is nevertheless exceedingly low.

Pre-Impact Model Compared to Post-Impact Model

	Pre-Impact Model	Post-Impact Model	Divergence Factor	
DART Spacecraft Mass:	570 kg	–	–	
DART Impact Velocity:	6258.5 m/s	–	–	
Change in Orbital Period:	-200 sec*	-1,938 ± 120 sec	~9.5X	Observed Change
Post-Impact Delta-V:	-0.5 mm/s*	-2.5 ± 0.1 mm/s	5X	Based on Post-Impact Orbital Period
Dimorphos Mass:	5.68 x 10^9 kg	1.33 x 10^9 ± 0.3 kg	0.2 – 0.3X	
Dimorphos Density (≈ Didymos):	2.17 g cm^3	0.6-0.7 g cm^3	0.3 – 0.4X ?	
Dimorphos Porosity (Void Space %):	~20-30%	≳70% ?	~3X ?	
Dimorphos Interior Structure:	'Same as Didymos'	Rubble Cloud ?	–	
Dimorphos Spherical Equiv Dia:	171 m	150 m ?	0.9X ?	Other Possible Factors
Momentum Transfer Efficiency:	60%	60-80% ?	1.0 – 1.2X ?	

*Consensus pre-impact predictions replicated by our model.

Table 1: Pre- and post-impact models compared.

Comparing the consensus pre-impact model with our post-impact model, Table 1 sets limits on the nature of the impact and its consequences.

In addition to an unexpectedly low Dimorphos density, two factors may contribute to the orbital period and delta-v divergence. Perhaps due to the unique geological nature of low-density targets [10], the momentum transfer efficiency of the Dimorphos impact was > 60%. Also, the volume of Dimorphos might be slightly less than predicted by pre-impact observations, suggesting a lower target mass and consequentially greater delta-v. However, even when we assume a reasonably greater transfer efficiency and a reasonably smaller Dimorphos volume, our application of the Rocket Equation strongly suggests a Dimorphos average density of ≲0.7 g/cm^3 and perhaps as little as 0.6 g/cm^3.

In view of Dimorphos' apparently rocky composition, this suggests an interior porosity ≳ 70% …not a rubble pile …instead, a tenuous *rubble cloud*.

Conclusions:

1. The Tsiolkovsky Rocket Equation is a suitable analytical method for the study of impact excited momentum exchange.

2. Dimorphos is a low-density object (≲0.7 g/cm^3) corresponding to an interior porosity of ≳70% …very likely, an unconsolidated gravity-bound cloud of debris.

3. The DART projectile efficiently accelerated a high porosity rocky target, suggesting that poorly consolidated asteroids may be suitable targets for spacecraft impact orbital deflection missions.

References: [1] Tsiolkovsky (1903) *The Science Review*. [2] JHUAPL (2022) https://dart.jhuapl.edu/News-and-Resources [3] Cheng et al. (2018) *Planet. Space Sci*., 157, 104-115. [4] Michel et al, (2022) *Planet. Sci. J.*, 3:160. [5] Melosh (1989) *Impact Cratering: A Geological Process.* [6] Ramsley and Head (2017) *Planet. Space Sci*., 138, 7-27. [7] Scheirich and Pravec (2022) *Planet. Sci. J.*, 3:163. [8] Naidu et al. (2020) *Icarus*, 348, 113777. [9] Nakano et al. (2022) *Planet. Sci. J.*, 3:148. [10] Schultz et al. (2007) *Icarus*, 105, 295-333.