SOME RESULTS OF THE 153 HILDA ASTEROID ORBIT APPROXIMATION. A. E. Rosaev[1], [1]Research and Educational Center "Nonlinear Dynamics", Yaroslavl State University, hegem@mail.ru

Introduction: The Hilda group is a unique array of orbits in the exact (3:2) mean motion resonance which is stable for a long time. In other resonances in the main belt, we have gaps that asteroids rarely visit. The origin of this group is connected with the process of planetary migrations [1]. For these reasons, a detailed study of the dynamics of this group is very important. The present paper provides some new quantitative data and draws attention to the difference in the dynamical state of the asteroids within the Hilda group.

The quantitative characterization of amplitudes and periods of perturbations in orbital elements can help to our understanding the minor bodies dynamics, especially in case motion in vicinity of resonances. The one of example is well known: harmonic (Fourier) approximation. However, it is valid only in a finite interval and significantly depends on the choice of the integration interval.

In our previous paper we applied another type of approximation by the sum of perturbations with different (incommensurable) frequencies [2]. However, not all resonance perturbations can be approximated by this approximation. Here we describe the other form of approximation, applicable in resonance case.

Methods: The evolution of the orbital elements of asteroids is obtained by numerical integration. To study the long term dynamical evolution of orbits, the equations of the motions of the systems were numerically integrated backwards 800 kyrs, using the N-body integrator Mercury [3]. To avoid short periodic perturbations, we averaged the results of numerical integration in a 500 year window. The initial epoch of our integrations was T_0=1998-Jul-06 (JD2451000.5).

Previously [2], we use the approximation by a sum of trigonometric terms with arbitrary frequencies:

$$E_i = E_{i00} + E_{i0}t + \sum_k^N \left(c_{ik}\cos(\omega_{ik}t + \varphi_{ik})\right) \qquad (1)$$

Here coefficients c_{ik} –amplitude, ω_{ik} - frequency and φ_{ik} - phase. The final approximation reached when the standard error which calculated using the expression:

$$\sigma_k = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(E_i - E_{iapprox})^2} \quad , \qquad (2)$$

when it becomes minimal.

The first possible application of our approximation is an independent (quantitative) estimation of the age of close asteroid pair and families. An example of this application is considered in our papers [4]. The second important application is the study different resonance perturbations in the evolution of orbital elements of asteroids. The first attempt of such study was done in paper [5]. However, not all resonance perturbations can be approximated by the expression (1). Here we describe the other form of approximation, applicable in resonance case.

Obviously, the more complex approximation can required. For the better fit such complex cases it is natural to apply the approximation by the combinational frequencies:

$$E_i = E_{i00} + \sum_k^N c_{ik}\cos(\omega_{ik}t + \varphi_{ik})\cos(\nu_{ik}t + \vartheta_{ik}) \qquad (3)$$

As above, the final approximation reached when the standard error which calculated using the expression (2) obtained minimal value.

Results: First, we have made the approximation of 153 Hilda orbital elements by method [2]. The method is successful for inclination and node longitude (t in kyr):

$$\Omega = 231.4° - 17.55t - 9.79\cos(0.3071t + 6.13) +$$
$$+ 2.13\cos(0.0057t + 5.85)$$

The standard error after the first (linear approximation is σ_1=6.85°, after the second approximation σ_2=2.65° and after the third ones σ_3=2.58°. The value 17.55 deg/kyr is equal to 63 arcsec/yr (in agreement with data in AstDys site).

For the inclination approximation we have:

$$i = 9.01° + 1.5595\cos(0.30695t + 4.3809) -$$
$$- 0.475\cos(0.1786t + 4.48)$$

The evolution of eccentricity, perihelion longitude and semimajor axis is complicated by short and long periodic perturbations. For better fit such complex cases it is natural to apply the approximation by the combinational frequencies. The evolution of the orbital elements of the (153) Hilda in the 3:2 resonance with Jupiter can be approximated by the expression (3), Fig.1:

$$e = 0.18 + 0.074\cos(0.0615t + 5.675) \times \cos(2.46t + 1.19)$$

$$\varpi = 245.0° - 136.35t + 25\cos(0.0585t + 5.55) \times \cos(2.46t + 4.0)$$

The value 136.35 deg/kyr is equal to 490.9 arcsec/yr (in agreement with data in AstDys site).

As it is seems, one of the perturbation periods in eccentricity is equal to ones in perihelion longitude. Similarly, the periods in inclinations and node longitude are the same.

For the approximation of the semimajor axis evolution we have obtained (Fig.2):

$$a = 3.9654 + 0.002\cos(0.055t + 1.1) \times \cos(2.32t + 3.0)$$

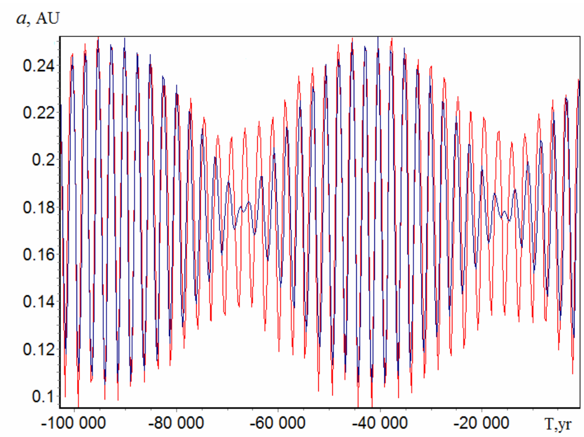

Fig.1 The eccentricity of the 153 Hilda evolution and approximation

To comparison, we applied the approximation of the orbital elements evolution for the 153 Hilda asteroid by Fourier frequencies. First, we use the 100 kyr interval for the Fourier approximation. In result we obtain the frequencies of main perturbations in eccentricity 2.39±0.03 kyr^{-1} and 2.51±0.03 kyr^{-1}. After that we use the 800 kyr interval for the Fourier approximation. It gives the better result. Finally, we obtain the frequencies of main perturbations 2.392±0.005 kyr^{-1} and 2.510±0.004 kyr^{-1}.

By our method we obtain the values 2.399±0.001 kyr^{-1} and 2.522±0.001 kyr^{-1} respectively. Our values are in range of errors obtained by Fourier method but more precise.

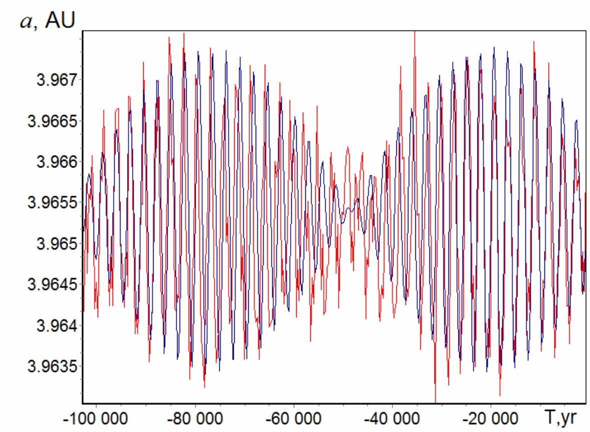

Fig.2 The semimajor axis of the153 Hilda evolution and approximation

Conclusions: The perturbation periods in eccentricity and in perihelion longitude are very close. Similarly, the periods in inclinations and node longitude are the same. The approximation of the 153 Hilda orbital element evolution is done over the 1 Myr interval. The eccentricity and perihelion longitude of the 153 Hilda is better approximated by combinational frequencies.

References:

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