SURFACE RELIEF RETRIEVAL WITH THE IMPROVED PHOTOCLINOMETRY METHOD: SOURCES OF ERRORS AND THE LOCAL ACCURACY. *N. V. Bondarenko[1,2], I. A. Dulova[2]*, [1]Earth and Planetary Sciences, University of California – Santa Cruz, Santa Cruz, CA, 95064, USA, nbondar@ucsc.edu, [2]Institute of Radiophysics and Electronics, NAS of Ukraine, 12 Ak. Proskury, Kharkiv, 61085, Ukraine.

Introduction: Obtaining topographic information from orbital images is widely used in planetary science; photoclinometry is a class of algorithms for such data processing. The improved photoclinometry method (IPM) is based on the Bayesian inference approach [5] and allows the most probable surface relief retrieval using images. Number of initial images is not limited. At least two images with different solar azimuths (the closer the difference between them to 90°, the better) are needed to define the height gradient.

In this work we discuss sources of errors inherent in the IPM, approaches for this method implementation and expected global / local accuracy of retrieved surface heights.

IPM and its implementation: Following "traditional" photoclinometry (initially proposed by Van Diggelen [1]), IPM involves the dependence of the surface facet brightness on its orientation. This priory known dependence allows recalculation of the observed reflectance field $J(x,y)$ into the slope field $\mathbf{t}(x,y)$. In the first order approximation IPM uses only two first terms of a Taylor series expansion of $J(x,y)$:

$$J_j(x,y) = J_{0j}+\mathbf{c}_j\mathbf{t}(x,y), \qquad (1)$$

where j – image number and J_0 – constant value.

Since all observations are accompanied with a noise of registration, the gradient of the true surface height $H(x,y)$ differs from image-derived slopes by a random value $\delta(x,y)$:

$$\nabla H(x,y) = \mathbf{t}(x,y) - \delta(x,y). \qquad (2)$$

We consider the true relief and the image noise N to be realizations of stationary Gaussian processes with spectral densities $I_H(\mathbf{k})$ and $I_N(\mathbf{k})$, respectively ($\mathbf{k}$ is a spatial frequency vector).

We use two approaches for IPM processing: IPM-F, calculations in the spatial spectral domain using the Fourier transform, and IPM-T, numerical solution of the Poisson equation with the finite difference method.

Taking into account that Fourier components of the stationary Gaussian process are statistically independent, IPM-F operates with heights' Fourier -components calculated for every $\mathbf{k}$ through the expression [3]:

$$\widetilde{H}(\mathbf{k}) = -i\mathbf{k}\widetilde{\mathbf{t}}\Big/\left(I_N / I_H + \mathbf{k}^2\right), \qquad (3)$$

where the tildes denote Fourier transforms of corresponding variables, and $(I_H/I_N)^2$ is the signal-to-noise ratio (SNR). The inverse Fourier transform of results obtained by (3) gives the most probable relief given the source images.

If a spectral density of noise δ is constant in the spatial frequencies' domain, the surface relief can be calculated through the Poisson equation with von Neumann boundary conditions [4], (IPM-T):

$$\Delta H(x,y) = \nabla\mathbf{t}(x,y). \qquad (4)$$

IPM-F implementation with the fast Fourier transform is computationally effective; however, it requires the image dimensions to be a power of two. It also requires knowledge of SNR. IPM-T can be applied to images of any size. It does not require SNR explicitly; instead, IRM-T requires the use of the most probable gradient values, which can be calculated using, for example, the least square method. IPM-T processing may take a long time due to an iterative procedure involved.

IPM sources of errors: Errors of heights determination γ with IPM according to [5] can be estimated for every spatial frequency vector $\mathbf{k}$ as

$$\widetilde{\gamma}(\mathbf{k}) = (P(\mathbf{k})-1)\widetilde{H}(\mathbf{k}) + \sum_j Q_j(\mathbf{k})\widetilde{N}_j(\mathbf{k}), \qquad (5)$$

where $P(\mathbf{k}) = A(\mathbf{k})\big/B(\mathbf{k}), \quad Q_j(\mathbf{k}) = i\mathbf{k}\mathbf{c}_j\big/B(\mathbf{k}),$

$$A(\mathbf{k}) = \sum_j (\mathbf{k}\mathbf{c}_j)^2, \;\; B(\mathbf{k}) = I_H(\mathbf{k})\big/I_N(\mathbf{k}) + A(\mathbf{k}).$$

The second term in (5), *R*, depends on particular noise realizations; it is a noise passed through the filter (3). The first term in (5), *SR*, is the error occurred due to smoothing the true relief by this filter. The *SR*-term in (5) is a part of the systematic error in heights determination. Contrariwise, *R*-term in (5) presents the random heights' error in the relief retrieval.

Other error sources affecting IPM-F are sampling of source images and their limited dimensions that lead to the limited spatial frequency range and periodically repeated images of the base spectrum in the Fourier spatial frequency domain, respectively. Von Neumann boundary conditions used in IPM-T instead of the true heights at boundaries is the main error source for this IPM processing type. IPM-retrieved relief has higher heights errors near boundaries for both IPM-F and IPM-T.

IPM-retrieved heights' errors: test calculations: We used two types of surface relief models: a lunar-like cratered surface (Fig. 1a) and an artificial hills / depressions relief (see an example in Fig. 1b).

The model in Fig. 1a has dimensions of 512×512 px; heights vary from $-2.95\sigma_0$ to $+3.13\sigma_0$, where σ_0 is the standard deviation of the model relief heights. Dimensions of the area in Fig. 1b is 1024×1024 px; it is a flat surface with hills and depressions having symmetric shapes of the same size L and the same absolute heights/depths H_F at the top/bottom (red circle in Fig. 1b). L is equal to 256 px in Fig. 1b.

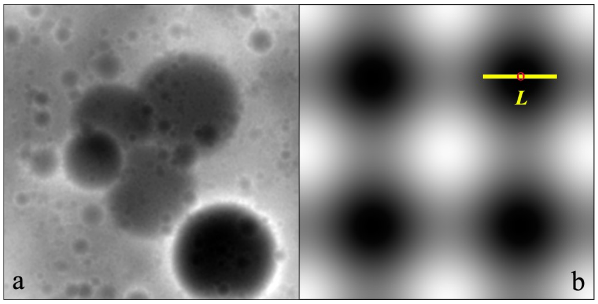

Fig. 1. Relief models: (a) a lunar-like relief; (b) an artificial hills / depressions relief, the yellow bar marks features diameter L, the red circle – the bottom of the depression. Darker shades correspond to lower heights.

For each type of the model relief (Fig. 1) we generated pairs of simulated initial images with solar azimuths normal to each other and SNR equal to 1, 10, 50, and 100. We assumed the Lambert photometric function and constant albedo over the whole area. Simulated images were used to retrieve surface heights using both IPM-F and IPM-T. For the model relief in Fig. 1a calculated heights root-mean-square errors ε_H are presented in Tab. 1. Tab. 1 shows retrieved heights relative to the source model relief for IPM-F and IPM-T implementations along with random R and total γ theoretically predicted errors according to eq. (5).

Data in Tab. 1 evidence that IPM-T yields more correct heights retrieval in comparison with IPM-F. Theoretical (5) γ values are the best possible accuracy of the relief retrieval by IPM-F. It is responsible for the main part of observed errors for SNR = 1, 10 and 50. IPM-F ε_H goes over γ values due to discussed above IPM-F properties, in particular, responsible for higher retrieved heights errors near boundaries. ε_H is much lower in the inner parts of the model domain (see IPM-F* and IPM-T* in Tab. 1).

The dependence of heights errors ε_H for small features on feature size L (the model relief like in Fig. 1b, SNR = 50, IPM-T) is shown in Fig. 2a. Size L varied from 4 to 512 px; t values 0.025, 0.224 and 0.432 ($t = |H_F|/L$; a measure of the feature "sharpness") correspond to blue, red and green curves, respectively. Height retrievals are least accurate for the smallest-scale features for all three cases. The best retrievals are for 16 – 32 px features and smoother surfaces.

The dependence of ε_H on an incidence angle θ is presented in Fig. 2b for several (L, t) combinations (the model relief like in Fig. 1b, SNR = 10, IPM-T). Smaller ε_H values are seen for smoother surfaces (see blue, red and green curves, L = 128 px). When the "sharpness" t of the surface is the same, ε_H values are close to each other for all feature sizes. For example, in Fig. 2b solid black, red and dash black curves correspond to t = 0.224 and L = 8, 128 and 256 px, respectively.

Table 1. Heights errors ε_H in terms of σ_0.

Type	SNR			
	1	**10**	**50**	**100**
IPM-T	0.2703	0.2640	0.2137	0.2237
IPM-F	0.8593	0.5599	0.3709	0.3308
R	0.0214	0.0077	0.0037	0.0026
γ	0.8318	0.4877	0.2165	0.1309
IPM-T*	0.1917	0.1675	0.1558	0.1621
IPM-F*	0.8259	0.4643	0.2525	0.2125

* denotes ε_H calculated over the central part (64 px margins excluded).

Better height retrievals occur for higher θ (Fig. 2b). On the other side, higher θ can cause shadows in images; the θ range supporting shading effect showed with the gray rectangle in Fig. 2b.

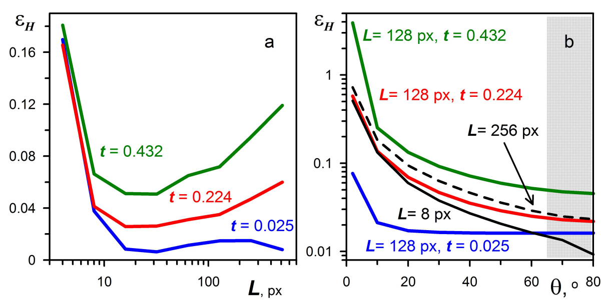

Fig. 2. IPM-T processing: heights errors ε_H as a function of (a) feature sizes L, SNR = 50; (b) incidence angles θ, SNR = 10, the θ range supporting shading effect showed with the gray rectangle.

Conclusions: IPM-T heights errors vary in the range of 0.21-0.27 σ_0, but small-scale features' local accuracy is much better and can be as low as 0.01-0.06 σ_0 (σ_0 – surface heights standard deviation). The farther away from the image boundaries, the more accurate heights retrieval, the lower heights errors.

References: [1] Van Diggelen J. (1951) *Neth. Astron. Inst. Bull., 11*, 283-289. [2] Bayes, T. (1958) *Biometrica, 45*, 293-315. [3] Kornienko, Yu. V. and Dulova I. A. (2019) *Radiofiz. Elektron.*, *24*, 46-52. [4] Kornienko, Yu. V. et al. (2021) *Radio phys. radio astron.*, *26*, 173–188. [5] Dulova I. A. and Kornienko Yu. V. (2001) *Radio phys. radio astron.*, *6*, 310–316.