TESTING A NUMERICAL MODEL FOR PAHOEHOE FLOW FIELD DEVELOPMENT.

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Introduction: Quantitative numerical models for the emplacement of simple and/or channelized lava flows have been in use for decades [e.g., 1-3]. These models are able to reproduce the general characteristics of active lava flows on Earth and are thus applied with some confidence to planetary lava flows [e.g., 4,5]. The same cannot be said for compound pahoehoe flowfields which make up a very large fraction of known lava flows on the Earth, Mars, and Io. The quantitative "modeling" of these flows has been largely restricted to simple thermal budgets [6,7]. When models for simple flows have been applied to compound pahoehoe flow fields, they have resulted in massively inaccurate results [8]. The goal of this study is to test if a novel numerical approach can reproduce the behavior of well-documented compound pahoehoe flow fields on Kilauea Volcano, Hawaii. If this testing were to be successful, the same model would be applied to similar flows on Io and then Mars.

The New Model: The central pillar of the new model is the hypothesis that compound pahoehoe flow advance is more akin to diffusion than classic fluid flow. This idea is born of many years of qualitative observations of active flows on Kilauea but is related to the "random walk" model presented by Baloga et al. [9]. This model only has value if (a) it adequately reproduces observed flow behavior and (b) is significantly simpler than attempting to model the full physics of compound pahoehoe flows. One major simplification is to model in two spatial dimensions and have flow thickness (h) be the quantity that diffuses. This is possible because a huge amount of complexity can, at least theoretically, be absorbed into the diffusivity term (κ) in the diffusion equation (Eq. 1). In principle, this mirrors the use of the Bingham rheology to model simple ʻaʻā flows. The yield strength term absorbs a huge amount of complexity related to the brecciated crust. Bingham models for lava flows can be useful (within limits) despite that fact that lava does not have a Bingham rheology.

$$\frac{\partial h}{\partial t} = \kappa \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) \qquad (1)$$

We developed the criteria that would define success before starting the test. The primary goal was to observe the development of preferred pathways that evolve into stable lava tubes over a timescale of weeks to months in flows a few meters thick given an effusion rate of a few m^3/s. A second major test was to have the flow front behave broadly correctly. The flow should be wider on shallow slopes and narrower on steeper slopes. For an effusion rate of ~4 m^3/s (typical of the activity on Kilauea in the 1990s), the flow should have a width of kilometer-scale on slopes ~1 percent and hectometer-scale on slopes ~10 percent. The flow front should advance hundreds of meters per hour on steeper slopes but only tens of meters per hour on very shallow slopes.

Numerical Methods: Rather than develop a new numerical model, we have relied on the commercial COMSOL Multiphysics software package. However, rather than use the pre-existing thermal or chemical diffusion modules, we found it more effective to use COMSOL's "equation" module. This allows the partial differential equation (PDE) to be tackled directly without limitations imposed by the specific problem the different modules were designed to address.

The equation module solves an extremely generic PDE but we only utilized the terms related to time-dependent diffusion in two dimensions. This meant that all the physics was tied to just the diffusivity (κ). κ needed to be (1) sensitive to slope, (2) have a positive feedback mechanism for areas where flow was enhanced, and (3) include a negative feedback mechanism for areas where flow was reduced.

We included a slope effect by having κ proportional to the slope squared. A nuance was that we used the "effective" slope (θ) of the flow of the top of the flow, not the base. The effective slope goes to zero as an area is filled with lava.

The positive feedback was provided by having κ proportional to the square of the flow thickness (h). This had the benefit of making the flow front convex up, rather than concave up as is typical of diffusion problems.

The negative feedback loop was created by having crust growth that removed a portion of h from consideration in calculating κ. The rate of crust growth was taken from field observations in Hawaii [10] but reduced where flow was vigorous (i.e., κ was high). This "effective" melt thickness is denoted h_m.

Combining these three effects in equation form gives

$$\kappa = C_1\, \theta^2\, h_m^2; \quad h_m = C_2\, t^{1/2} / (1 - C_3 \kappa) \qquad (2)$$

C_1 is proportional to gravitational acceleration and the density of the lava. C_2 is 7×10^{-7} m^2/s taken from the empirical crust growth model from [10] and C_3 is not well-constrained by observations but was set to 0.1 which provides about an order of magnitude variation over the expected range of κ. Slope was included as a polynomial fit to the section of the Hilina Pali that was traversed by multiple pahoehoe flow fields in the 1990s.

Preliminary Results: Given the multiple feedback loops, this problem is prone to numerical instabilities. It was necessary to ease into the initial conditions over a period of 1 second of model time. Various if-statements had to be added to avoid divide by zero at the start. To maintain a steady flux, the depth of lava at the vent had to be steadily increased but there was no *a priori* way to calculate what this flow thickness should be. An estimation scheme had to be tuned by iteration to provide the correct flux at the start of the actual run (1 second in model time) and then provide approximately correct values for the growth rate in vent depth as a function of vent depth. This approximate value was then fine-tuned based on the degree of excursion from the desired value. The process was iterated until lava flux was held to within 1 percent of the desired value.

COMSOL provides powerful tools to build meshes of nodes for the numerical computations and has sophisticated algorithms to allow large timesteps without losing accuracy or stability. However, even with these capabilities, it was not possible to use meshes that were coarser than 1 meter between nodes without introducing serious numerical artifacts. This was especially problematic in the sector where the lava flowed upslope from the vent. While the equations properly address this case, avoiding numerical artifacts proved very computationally expensive. It was more expedient to ignore this small portion of the flow and only consider lava that moved downslope from the vent.

Initial runs showed that for C_1 of ~1000 the lava spread at a reasonable rate and produced flows of a reasonable thickness. This formulation led to higher diffusivity on steeper slopes. This had the desired effect of having the flow extend preferentially in the downslope direction but the undesired effect of making the flow broader on steeper slopes. To address this, diffusivity was made non-isotropic with diffusivity in a direction dependent only on the slope in that direction. This creates the desired behavior as a function of slope: lava extends down the steep part of the slope in a couple of days and then dramatically slows on the coastal flats, causing flow to back up and inflate. The crust formation is also realistic. The crust grows at the expected rate other than where it is retarded by rapid flow in the proto-lava tube.

Conclusions: Two points are amply clear from this testing: (1) the diffusion model for compound pahoehoe flow field emplacement has the ability to reproduce many of the key aspects of real flow fields, *but* (2) the computational cost is extremely high and this model will not provide a simple, easy to use, tool akin to the Bingham model for simple ‘a‘ā flows. The complex 3-part definition for diffusivity does not provide a simple parameter akin to “yield strength” in the Bigham model. Given this situation, we conclude that the better path is to use a numerical approach that explicitly includes each of the multiple physical processes that are all conflated into diffusivity in the current approach. This model is in the unusual and unfortunate position of being (largely) correct but not useful.

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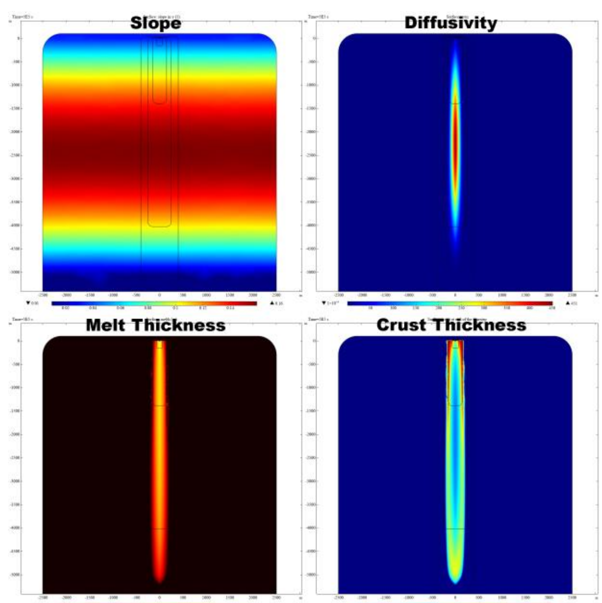

Figure 1. Example output from COMSOL for simulated pahoehoe flow field extending down the Hilina Pali on Kilauea Volcano, Hawaii. Output after 10^5 seconds (1.16 days) of model time.