

SLOPE STABILITY ANALYSIS OF TOP-SHAPED RYUGU. N. Namiki^{1,2}, T. Mizuno^{2,3}, H. Senshu⁴, H. Noda^{1,2}, K. Matsumoto^{1,2}, N. Hirata⁵, R. Yamada⁵, Y. Ishihara⁶, H. Ikeda³, H. Araki^{1,2}, K. Yamamoto¹, S. Abe⁷, F. Yoshida⁸, A. Higuchi⁸, S. Sasaki⁹, S. Oshigami³, S. Tsuruta¹, K. Asari¹, S. Tazawa¹, M. Shizugami¹, H. Miyamoto¹⁰, H. Demura⁵, J. Kimura⁸, T. Otsubo¹¹, N. Hirata¹², G. Nishiyama¹⁰, F. Terui³, S. Watanabe¹³, T. Saiki³, S. Nakazawa³, M. Yoshikawa³, and Y. Tsuda³, ¹National Astronomical Observatory of Japan (2-21-1 Osawa, Mitaka, Tokyo, Japan 181-8588, nori.namiki_AT_ao.ac.jp), ²The Graduate University for Advanced Studies, SOKENDAI, ³Japan Aerospace Exploration Agency, Institute of Space and Astronautical Science, ⁴Chiba Institute of Technology, ⁵The University of Aizu, ⁶National Institute of Advanced Industrial Science and Technology, ⁷Nihon University, ⁸University of Occupational and Environmental Health Japan, ⁹Osaka University, ¹⁰The University of Tokyo, ¹¹Hitotsubashi University, ¹²Kobe University, ¹³Nagoya University.

Introduction: C-type asteroids have been considered as a possible transporter of water and organic matters to inside of snow line at the time of the solar system formation. Their capacity of transportation, however, depends on vertical advection of regolith and boulders inside of the asteroid, because volatiles can be quickly removed from the outcrop by a heat of solar radiation. Besides, successive discoveries of rubble pile asteroids [1, 2, 3] make this problem more complicated because granular flow in micro gravity environment has not been understood well yet.

The shape of rubble pile asteroids can provide a clue to this problem. Previously the ellipsoids were regarded as the likely equilibrium figure of rotating self-gravitating aggregates [e.g. 4, 5, 6] while in situ observation of Hayabusa2 and OSIRIS-REx have revealed top-shape of small asteroids [2, 3]. The formation process of the top shape is still under debate [e.g. 7, 8, 9, 10], and it is not clear yet whether the mass transfer is constrained in the surficial layer or is involved in internal deformation.

Slope Stability Analysis: In the field of soil mechanics, stability of the slope is evaluated simply by a ratio of traction along the slope and resistance caused by friction and cohesion, that is, a factor of safety, FoS [e.g. 11]. In this study, we use an inverse of FoS because the traction may become zero depending on rotation period, P .

$$FoS^{-1} = \frac{T}{N \tan \phi}$$

where ϕ is an angle of repose. T and N are traction and normal forces, respectively.

$$T = m (-g_R + R (2\pi/P)^2 \sin \theta) \cos(\theta - \phi) + m g_z \sin(\theta - \phi)$$

$$N = m (g_R - R (2\pi/P)^2 \sin \theta) \sin(\theta - \phi) + m g_z \cos(\theta - \phi) + C_0 S$$

where m is a mass of rock at the surface, S is its area adjoining the surface, g_R and g_z are gravitational attractions in radial and rotation-axis directions, R and θ are the radius and colatitude from the center of Ryugu, ϕ is the surface slope, and C_0 is cohesion stress. Conventionally C_0 works in the direction parallel to the

slope, however, we assume that C_0 is an attraction between rock and surface materials below.

Sand or soil cannot withstand for $FoS^{-1} > 1$ and will collapse. For $FoS^{-1} < 1$, the slope is metastable, meaning the slope can sustain unless external shaking is excited. $FoS^{-1} = 1$ is critical and the slope equals the angle of repose.

The “top shape” of asteroids is often referred in the literatures, but has not been well quantified. Following [12, 13], we simplify the top shape of Ryugu by taking even zonal, i.e. axisymmetric and north-south symmetric, components of spherical harmonics expansion of shape model [2]. While the original spherical harmonics are expanded up to degree 180, high frequency components cause unstable oscillation near poles. Then we truncated at degree 30 where overall topography of top shape is well reproduced and artificial oscillation is negligible.

Gravitational attraction at the surface of top shape is approximately calculated by 18,000 disks of constant surface density. The top shape is divided into 180 truncated cones whose thickness is 1 degree in latitude. The gravity potential of each truncated cone is then represented by 100 disks.

Formation of Top Shape: The shape of Ryugu appears best explained by [2] who calculate slope of topography with respect to local gravity for various spin periods, and find that a histogram of slopes approaches to Gaussian distribution when P is 3.5 hr. The average of the slope is 31°, and this value is interpreted as an angle of repose of Ryugu regolith. Similarly, we assume that the current shape of Ryugu was critical in the past fastest rotation. Thus we set $FoS^{-1} = 1$ and calculate $C_0 S/m$ as a function of θ for given P and ϕ .

Figure 1 shows such calculated $C_0 S/m$. Considering that $C_0 S/m$ and ϕ shall be independent of θ , plausible sets of P and ϕ are constrained $C_0 S/m$ is constant regardless of θ . Light blue line corresponds to the present P of 7.63 hr and ϕ of 40° which is usual for terrestrial sand. This set results in negative, namely, repulsive $C_0 S/m$ meaning slopes are globally metastable at present. Following [2], we also calculate

for a set of P of 3.5 hr and ϕ of 31° (orange line). The result seems close to a constant C_0S/m , at least for a range between 20° and 65° while there is a room of improvement. In this figure, the set of P of 3.85 hr and ϕ of 24° gives the least deviation of C_0S/m for θ between 20° and 65° . A wavy variation is probably due to truncation of spherical harmonics at degree 30.

We have continued the same analyses for ranges of P and ϕ (Figure 2). A condition for plausible P and ϕ is arbitrary set such that the standard deviation of C_0S/m is less than $5 \times 10^{-6} \text{ m s}^{-2}$ (area bordered by orange dashed line). Consequently, P , ϕ , and C_0S/m that match the present top shape are constrained to be between 3.8 and 4 hr, 20° and 25° , and less than $2.5 \times 10^{-6} \text{ m s}^{-2}$, respectively.

Calculated P and ϕ are longer and smaller than the previous estimates [2], but is greater than $17 \pm 2^\circ$ of Bennu [12, 14]. And the upper bound of C_0S/m is about 18 % of the surface gravitational attraction. This value is small, but is not negligible for surface shedding [5, 15] if P was fast in the past. Taking the rock density of 1380 kg m^{-3} [16], our estimate results in $C_0 = 0.006 d \text{ Pa}$ where d is a diameter of rock in m. Application of size-dependent cohesion of lunar regolith [17] yields d of about 1 m as a critical size of rocks that moves at Ryugu surface.

Latitudinal Spectral Variation: Spectral slopes at poles and equator of Ryugu show distinctive blue characteristic [18] indicating mass wasting from the equator and poles to mid-latitude [19]. We have examined such spectral slope variation in the view of slope stability with an increase of P . Figure 3 shows FoS^{-1} as a function of θ and P . C_0S/m and $\tan\phi$ are assumed to be $1.9 \times 10^{-6} \text{ m s}^{-2}$ and 22° , respectively, from the results in Figure 2. As Ryugu spins down, critical slope becomes metastable, however, FoS^{-1} remains positive in the most region suggesting rocks

tend to move in a direction from poles to equator. Only equatorial region becomes negative.

FoS^{-1} in the equatorial band can be negative depending on P , but is greater than -1 even at present. Thus, spin down solely cannot explain blue spectrum, of the surface of equatorial bulge, but an additional excitation is required. Probably its source is impact cratering. There are many large craters that postdate the equatorial bulge [20]. Latitudinal width of blue spectral slope appears $\pm 10^\circ$ [19] which corresponds to the orange line in Figure 3. Therefore the time of large impacts are likely to have occurred when P was 4.5 hr.

In conclusion, the top-shape of Ryugu is simply described by a force balance between traction along slope and friction due to normal force. The angle of repose is estimated to be between 20° and 25° .

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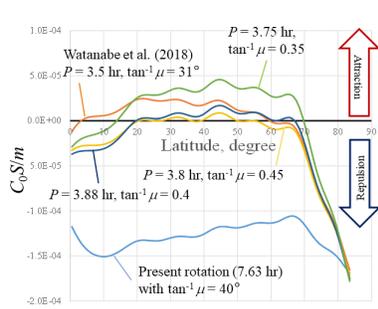


Figure 1. Cohesion required for critical slope. P and μ are assumed rotation period and internal friction ($\tan\phi$). Negative cohesion indicates that local slope is metastable without any cohesive forces.

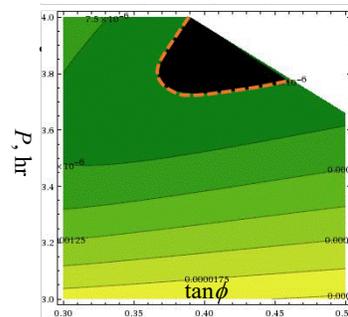


Figure 2. Standard deviation of C_0S/m over the range between 20° and 65° in latitude for variable P and $\tan\phi$. Sets of P and $\tan\phi$ which result in negative C_0S/m are eliminated from the figure (a white area at the upper right corner).

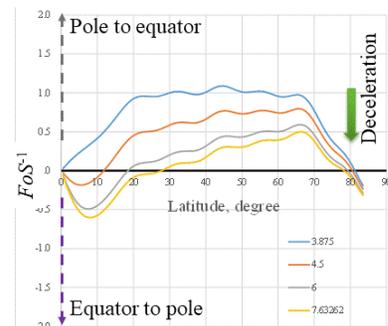


Figure 3. Changes of FoS^{-1} with increasing P . Light blue, orange, gray, and yellow lines are calculated for P of 3.875, 4.5, 6, and 7.63 hr, respectively.