Estimates of Martian mean recharge rates from analytic groundwater models. M. A. Shadab[1,2,†], E. Hiatt[2,3,4,*], and M. A. Hesse[1,2,4,‡], [1]Oden Institute for Computational Engineering and Sciences, [2]Institute for Geophysics, [3]Department of Geological Studies, Jackson School of Geosciences, [4]Center for Planetary Systems Habitability, The University of Texas at Austin, Austin TX ([†]mashadab@utexas.edu, [*]eric.hiatt@utexas.edu, [‡]mhesse@jsg.utexas.edu).

Introduction: We develop an analytic solution for a steady unconfined groundwater aquifer beneath the southern highlands of Mars and use it to explore self-consistent combinations of mean recharge and mean hydraulic conductivities. Our results show that due to the large difference between surface area and cross-sectional area of the aquifer only a comparably small amount of recharge is required to raise the water table to the mean land surface for all suggested shorelines of a hypothesized ocean in the northern lowlands.

Aquifer model: Similar to other Mars groundwater models [1-3], we use the Dupuit-Boussinesq model [4, 5] for the elevation, h, of the groundwater table above the base of the aquifer

$$\phi\frac{\partial h}{\partial t} - \nabla\cdot(Kh\nabla h) = r.$$

Here, ϕ and K are the mean porosity and mean hydraulic conductivity of the aquifer and r is the mean recharge. For this analysis, we assume that ϕ, K and r are constant, but our analysis can be extended to variable properties. The divergence and gradient operators take the standard form in spherical shell coordinates. We assume the flow is azimuthally symmetric so that the solution is only a function of the southern co-latitude, θ.

The boundary conditions are no-flow due to symmetry at the south pole, $\theta = 0$, and the elevation of the presumed ocean, h_o, in the northern lowlands at the shoreline, θ_o. We assume a crustal aquifer with a depth d of 10 km and a base at -9 km elevation. Further, we assume a simple step function topography with elevations of -4 km and 1 km in the lowlands and highlands, respectively (Fig. 1a). The change in topography occurs at, θ_o, the average southern co-latitude of the assumed shoreline. Here, the groundwater is in contact with the hypothesized ocean. We explore three different potential shorelines: Deuteronilus, Arabia and Meridiani with properties given in Table 1.

Shoreline	z [m]	h_o [m]	θ_o	A_{dry}
Deuteronilus	-3792	5208	132°	0.84
Arabia	-2090	6910	110°	0.67
Meridiani	0	9000	94°	0.53

Table 1: Properties for different shorelines used in computations. The mean elevations, z, are taken from [6]. The height of shoreline above the base of the aquifer is h_o. The co-latitude of the mean shoreline θ_o is computed from the fraction of the surface not covered by the presumed ocean, A_{dry}.

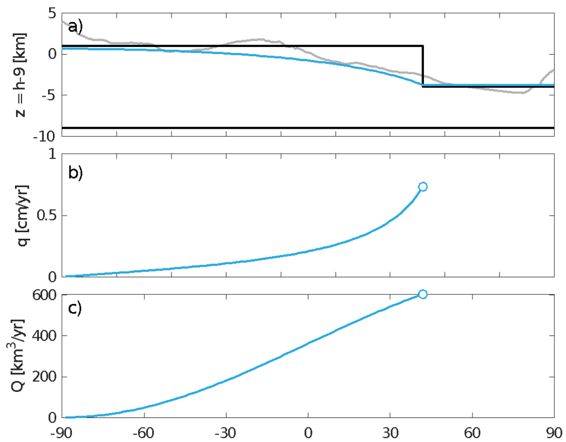

Figure 1: Analytic solution for steady unconfined aquifer on spherical shell with $h_o = 5208$ m, $\theta_o = 132°$, $K = 10^{-7}$ m/s, $r = 5\cdot 10^{-6}$ m/Earth year and $R = 3{,}389.5$ km. a) Head shown in blue (plotted as elevation) together with Mars' mean topography (gray) and the assumed step profile (black). b) Specific discharge, q. c) Total discharge, Q. The dots in panels b and c are from total mass balance over the aquifer.

Results: The analytic solution presented below is a unique opportunity to interrogate the relationship between the mean recharge and the mean hydraulic conductivity in the southern highlands aquifer.

Analytic solution. We explore the steady solution of the Dupuit-Boussinesq equation on a spherical cap with radius R. The elevation of the groundwater table above the base of the aquifer is

$$h(\theta) = \sqrt{h_o^2 + 2\frac{rR^2}{K}\, ln\left(\frac{\cos\theta + 1}{\cos\theta_o + 1}\right)}.$$

The specific discharge, $q = -K\nabla h$, is given by Darcy's law and varies with latitude as

$$q(\theta) = \frac{rR}{h(\theta)}\left(\frac{\cos\theta - 1}{\sin\theta}\right).$$

The total discharge, Q, of the aquifer as function of latitude is

$$Q(\theta) = h(\theta)l(\theta)q(\theta) = rA(\theta),$$

where $l = 2\pi R\sin\theta$ and $A = 2\pi R^2(1 - \cos\theta)$ are the length of the small circle and the area of the spherical cap corresponding to θ, respectively. The longitudinal variation of these three quantities is shown in Fig. 1 for the case of the Deuteronilus shoreline (Table 1). The effect of spherical geometry is evident in the total discharge (Fig. 1c), which increases most rapidly at the equator where the surface area and hence the azimuthally integrated recharge are the largest. The specific discharge sees the largest increase in the northern hemisphere, because the cross-sectional area of the aquifer decreases rapidly (Fig. 1b). Mass balance of the entire aquifer shows that the total discharge at the shoreline is $Q(\theta_o) = rA(\theta_o)$ and the specific discharge is $q(\theta_o) = Q(\theta_o)/(h_o l(\theta_o))$. These values are shown as dots in Figs. 1b and 1c.

Solutions for the groundwater table with all three shorelines and multiple values of recharge are plotted in Fig. 2. With decreasing elevation of the shoreline, the boundary, θ_o, between the groundwater and the ocean moves to the north (Table 1). Here, the Arabia shoreline in Fig. 2b requires the most recharge to raise the groundwater table to the surface. This is due to two competing effects. As the elevation of the shoreline decreases, the aquifer thickness decreases, but the total recharge increases. Here, we have chosen a mean hydraulic conductivity of 10^{-7} m/s and the figure shows that recharge fluxes on the order of 10^{-6} m/yr are sufficient to raise the groundwater table to the surface in all cases.

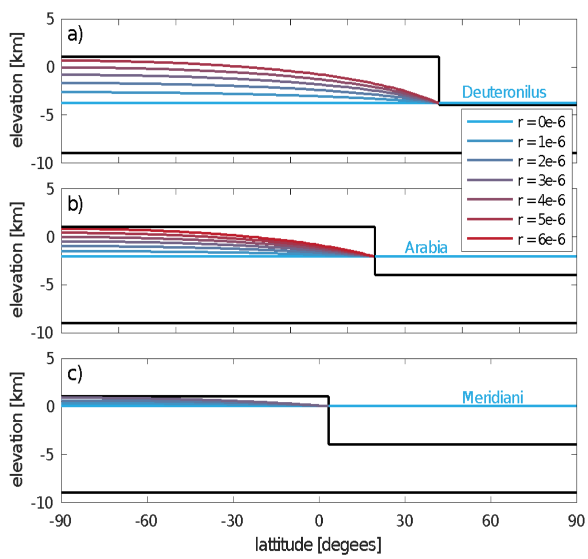

Figure 2: Solutions for the head for the three different shorelines given in table 1 with $K = 10^{-7}$ m/s, $R =$ 3,389.5 km and increasing values of recharge r in m/Earth year.

Mean recharge and conductivity of the highland aquifer. Neither the mean hydraulic conductivity nor the mean recharge of the southern highlands aquifer are known. However, we can use our analytic results to find combinations of conductivity and recharge that result in reasonable groundwater tables. We introduce the depth of the water table beneath the highlands, $D = d - h(0)$, evaluated at the south pole. Substituting this into the expression for $h(\theta)$ we obtain a quadratic equation for $D = D(K, r)$. Figure 3 shows the depth of the water table for the Arabia shoreline (Table 1). The maximum recharge is obtained when $D = 0$, i.e., the water table reaches the mean highland surface, and is given by

$$r_{max} = \frac{K(d^2 - h_o^2)}{2R^2 ln\left(\frac{2}{\cos\theta_o + 1}\right)}.$$

This provides the diagonal line that bounds the feasible solutions in Fig. 3. In the limit of small recharge and high conductivity the groundwater table approaches the sea level so that $D \rightarrow d - h_o$ (3090 m) in the bottom right corner. Finally, the maximum allowable recharge decreases with increasing surface area of the highlands, given by the radius, R, and the location of the shoreline θ_o.

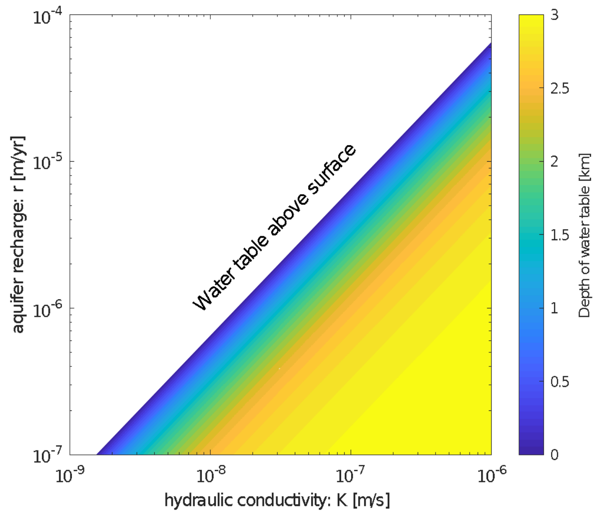

Figure 3: Water table depth, D, for the Arabia shoreline.

Discussion: Our results show that the mean recharge that can be accommodated by the southern highlands aquifer, without widespread seepage, is orders of magnitude smaller that published estimates of precipitation [7] and aquifer recharge [8]. Our results also show that very small amounts of recharge can effectively elevate the groundwater table. This is due to the very large surface area of the southern highlands relative to the cross-sectional area of the aquifer.

References: [1] Clifford (1993) *JGR, 98(E6),*10973-11016. [2] Hanna et al. (2005) *JGR, 110(E1).* [3] Luo and Howard (2008) *JGR, 113(E5).* [4] Dupuit (1863) *Dunod*. [5] Boussinesq (1904) *J. de mathématiques pures et appliquées, 10,* 5-78. [6] Carr and Head (2003) *JGR, 108(E5).* [7] Wordsworth et al. (2015) *JGR, 120(E6),* 1201-1219. [8] Andrews-Hanna et al. (2010) *JGR, 115(E6).*