Introduction: All planetary scientists are familiar with the problem of representing curved or complex volumes in two dimensions, i.e., on a map. The problem grows when the interest is global, as in hemispheric dust transport on “rubber ducky” comet 67P. We use our constant-scale natural boundary method (CSNB) [1] to make a pair of global maps with complementary perspectives on 67P’s south-to-north dust transport.

The maps are hand-plotted by an architect inspired by anamorphism, a topographic paper by James Clerk Maxwell and its later extension by Marston Morse [2], and poet Robert Frost’s love of accuracy in uses of the imagination [3]. CSNB has advantages over convention, e.g., it can include 67P’s Hathor region, an extreme overhang unmappable in standard projections.

A global phenomenon is hard to ponder because it is per se nonlocal: you can’t see it all at once. Global surfaces, theories, or events exist in non-Euclidean bounded 2-sheets. Comprehension with multiple regional maps, 3-D images or digital video is difficult because, per Miller’s Law [4], our short-term memory overfills; the mind gets stuck. Digital modes abandon the kinesthetic sense; conventional projections offer only perfunctory overview. CSNB, however, gives pictorially clear-cut (if necessarily non-unique) answers to non-Euclidean questions such as “What shape is Mars’ global watershed [5]?”

67P’s material transport is a high cartographic challenge because of its semi-orbital nature and its extraordinary irregularity. In a library without our method, scientists are best served by the digital arts, yet the intellect craves something unity and apprehensible and pertinent; CSNB provides it.

History of natural boundary mapping: Modern methods have evolved from algebraic abstractions of projection geometry and developable surfaces but the underlying concept—the Renaissance idea of perspective, ingeniously repurposed by Mercator in 1569—remains unaltered by later improvements [6].

The invention of artificial satellites, notably Landsat in 1972, provoked the chemist and cartographer J.P. Snyder to create the space oblique Mercator projection [7]. Snyder also worked with polymath A.F. Spilhaus on another innovation: world maps with natural boundaries. They established that such maps have an advantage in principle over other maps, and first voiced the idea of maps edged by continental divides, but didn’t accomplish it [8].

A more topological mapping paradigm: We extend Spilhaus and Snyder’s ideas by abandoning the xy “sheet goods” paradigm set by Edward Wright in 1599 [6] in favor of a simpler model: a 1-dimensional tree defining the 2-sheet interruption, coupled with a constant-scale map edge. Ergo, a 400-year orthodoxy of precise and unambiguous coordinate points enclosed by an amorphous border becomes an amorphous collection of point sets within a precise and unambiguous border. This nudges the math from geometry to topology, specifically prototopology, a term coined in 1943 by E. Panofsky to describe Albrecht Dürer’s 16th-century developments of Archimedian solids [9].

Our approach begets maps vested in topological parameters and physical features, with great liberty to include appropriate and exclude irrelevant features as the map edge. Unlike Spilhaus-Snyder, our maps resemble Dürer’s work: they fold to closed volumes.

Background: In 2016 we mapped 67P with a ridge-hugging edge. Pertinent linear features were traced from a physical model. Portions of traverses were assembled into map-sections, adjusted proportionally, and gratificed. The map folded to a volume within which the comet’s shape fit snugly, indicating that local shape distortion was minimal [10]. But our map poorly served the study of surface modification driven by migration of sublimated material from a warm, sun-facing side to a cold, opposing side. We wanted a map encircling the receiving hemisphere with the wasting hemisphere—north surrounded by south—and its logical complement. In 2017 we sketched such maps, using edges of geomorphological districts [11]. In 2018 we drafted them [12]. From feedback that mass wasting occurs in two directions from Ash, with Imhotep as a secondary trap, we adjusted [13].

Preserving relative proportions was challenging: Our maps were sub-optimal and in 2019 we corrected these deficiencies, checking cross-map ratios with five measurements rather than the two that had sufficed for Itokawa and Ida [14]. We used Tissot ellipses [15] to quantify local distortion and guide proportional restoration via “pruning the tree” in the neck. Scale increased in this area but our problem of clarity was solved. In 2020 we transformed photomosaics into our formats, giving the first comprehensive and logical looks at the comet’s surface (Fig. 1 & 2), from the point of view of hemispheric dust transport [16].

Results: This year, we refine the map-pair by selectively pruning or growing their respective trees to best conform to the précis. We also correct inaccuracies that crept in with tree adjustments by replotting the maps with a larger and more accurate model.
Discussion: Rosetta results show that the ~4 km by ~6 km object has complex morphology on cm to km scales. Its shape is bimodal; its origin, as two low-velocity-collision objects, is indicated by terrrace features (exposed layers of partially stripped icy volatiles), oriented differently on each mode. Cometary nuclei have transient morphologies. Change of 67P’s nucleus results from differential loss of gas and solid material due to solar-induced heating, varying as a function of solar distance and unbalanced seasonal exposure [17, 18, 19]. The highly porous, irregular surface indicates volatile removal via a combination of explosive release and sublimation. While much of this material escapes to become the tail, much also settles onto the opposite hemisphere.

Map versus physical model: The only accurate representation of an essentially globular object is an actual physical model; no computational simulation or global map can substitute completely for a full sensory appraisal, e.g., only by engaging the kinesthetic sense—holding a model—is it apparent that Aten and Khonsu are roughly parallel. The map—any map—is only an instrument of understanding.

Future: We welcome mosaics and suggestions to improve trees. We invite software collaborators.

Limitations: CSNB is a graphic experiment differing radically from digital transmutations of analytic projections. First glances can be disorienting, though this is often shortlived. Results are proof of concept, suitable for contemplation and communication.

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