FORECASTING FOR THE HAZARDOUS ASTEROIDS APPEARANCE. N. I. Perov1,2. 1State Autonomous Cultural and Education Organization named after V.V. Tereshkova. 150000, Yaroslavl, Ul. Chaikovskogo, 3, Russian Federation. E-mail: perov@yarplaneta.ru, 2State Pedagogical University named after K.D. Ushinskii. 150000, Yaroslavl, Ul. Republikanskaya, 108, Russian Federation.

Introduction: For unperturbed motion of the Solar system’s bodies there is collection of modes for the determination of epochs, corresponded for the minimal distances between the two bodies [2]. These arbitrary unperturbed trajectories may be determined, for example, from optical observations with using one and the same algorithm without singularities [3].

Below in the frame of the pairwise two body problem (“the Sun and a particle” and “the Sun and the Earth”), with taking into account the motion of line of apsides and line of nodes [1], the minimal distance between the Earth and a small body is determined.

The Based Equations: Let’s denote $a_1, a_2$ are the semimajor axes of the orbits of the Earth and the asteroid; $e_1, e_2$ are the eccentricities of the considered orbits; $i_1, i_2$ are the inclinations of the orbital planes in respect of the ecliptic plane for the given epoch; $\omega_1, \omega_2$ are the arguments of the perihelia of the Earth and the asteroids; $\Omega_1, \Omega_2$ are the longitudes of the ascending nodes of the considered planes; $v_1, v_2$ are the true anomalies of the bodies [1], [4], [5].

The relationship for searching of minimal distance $r_{12}$ between the Earth and the small body we represent in the form (1)

$$r_{12}^2 = (r_1 - r_2)^2 = \text{min.}$$

Here, $r_1$ and $r_2$ – the heliocentric vectors of the positions of the Earth and the asteroid.

In line with the theory [1], the following assumptions were made.

$$r_1 = \frac{p_1}{1 + e_1 \cos(v_1)}, \quad p_1 = a_1(1-e_1^2), \quad x_1 = r_1(\cos(v_1+\omega_1)\cos(\Omega_1) - \sin(v_1+\omega_1)\sin(\Omega_1)\cos(i_1)), \quad y_1 = r_1(\cos(v_1+\omega_1)\sin(\Omega_1) + \sin(v_1+\omega_1)\cos(\Omega_1)\cos(i_1)), \quad z_1 = r_1(\sin(v_1+\omega_1)\sin(i_1)).$$

The same equations (2)–(6) we put for the second body, using index “2”.

In order to take into account the variations of the quantities of $\omega_1, \omega_2, \Omega_1, \Omega_2$, we represent these parameters in the forms

$$\omega_1 = \omega_{10} + k_1 d_1, \quad \omega_2 = \omega_{20} + k_2 d_2, \quad \Omega_1 = \Omega_{10} + K_1 d_1, \quad \Omega_2 = \Omega_{20} + K_2 d_2.$$ 

Here $k_1, k_2, K_1, K_2$ are empirical constants. $d_1, d_2$ are variable values, with imposed a constraint on the ascending nodes (The closest approaching of the considered bodies takes place in the ascending nodes, which are equal).

$$\Omega_1 = \Omega_2,$$

or

$$d_2 = (\Omega_{10} + K_1 d_1 - \Omega_{20})/K_2.$$ 

The index “0” references to the initial data sets of $\omega_1, \omega_2, \Omega_1, \Omega_2$.

The quantities of $r_{12}$, $x$, $y$, $z$, and the first derivatives we find with using equalities (1) – (12) and the based equations we write as (13)

$$\frac{dv_{12}}{dv_1} = 0; \quad \frac{dv_{12}}{dv_2} = 0; \quad \frac{dv_{12}}{dd_1} = 0.$$

For the process of calculating we use the system “MAPLE’15”.

Example: Let’s find the numerical values of $v_1, v_2$, and $d_1$ for the epoch of the Earth and the asteroid 2019 SU3 approach. The initial orbital elements for the Earth and the asteroids are referred to the date 27 April, 2019 [1], [4], [5]. The unit of length is 1 AU. We put mass of the asteroid is less than mass of the Earth.

Using the empirical relation of $\omega$ to $\Omega$ for the planets of the Solar system [1] we may eliminate some singularities for the computations (for instance, if resonance in motion of the bodies is possible). So, put

$$k_1 = \sqrt{2}, K_1 = -1, k_2 = \sqrt{3}, K_2 = -1.$$ 

Numerical and analytical investigations of two body’s motion along heliocentric orbits in the interval of the several revolutions show there are local and global minimums of the distance $r_{12}$ between the bodies. (Fig. 1).
The distance \( r_{12} = r_{12}(v_1, v_2) \) between the Earth and the asteroid 2019 SU3 is measured in AU.

\[ d_1 = 0.0173501769 \text{ rad.} \]

The true anomalies \( v_1 \) and \( v_2 \) take values in the interval from 0 to \( 2\pi \) rad.

The formulae (1) – (13) permit to determine the distance \( r_{12} \), and plot the graph of the function \( r_{12} = r_{12}(d_1) \). (Fig. 2), (Fig. 3).

**Fig. 1.** The distance \( r_{12} = r_{12}(v_1, v_2) \) between the Earth and the asteroid 2019 SU3 is measured in AU. \( d_1 = 0.0173501769 \text{ rad.} \) The true anomalies \( v_1 \) and \( v_2 \) take values in the interval from 0 to \( 2\pi \) rad.

**Fig. 2.** One of the local minimums of the functions \( r_{12} \) is the distance \( r_{12} = r_{12}(d_1) \) between the Earth and the asteroid 2019 SU3 is measured in AU. \( d_1 \) is measured in rad. \( v_1 = 0.05981311596 \text{ rad, } v_2 = 0.008946200265 \text{ rad.} \)

**Conclusion:** The advanced way gives scope for take into account the motion of perihelia and ascending nodes of celestial bodies. Estimation of the perturbations offers to determine the global minimum of \( r_{12} \) between the Earth and the hazardous bodies with the smaller error (Fig. 3).

**Fig. 3.** The global minimum of the function \( r_{12} \) (\( r_{12} \) is the distance between the Earth and the asteroid 2019 SU3 measured in AU). \( d_1 \) is measured in radians. \( v_1 = 1.6822015501976 \text{ rad, } v_2 = 5.80709569091 \text{ rad.} \)

**References:**