LARGE-SCALE THERMAL MODELING AT THE LUNAR SOUTH POLE. Samuel F. Potter[1], Norbert Schörghofer[2], and Erwan Mazarico[3], [1]University of Maryland Department of Computer Science (sfp@umiacs.umd.edu), [2]Planetary Science Institute, [3]NASA Goddard Space Flight Center.

Introduction: The small obliquity of the Moon creates unique illumination conditions at the south pole, resulting in cold traps and permanently shadowed regions. Incorporating the occluding effect of fine topographic details on scattering is important for high-resolution thermal and illumination modeling; reflections and re-radiation (and their higher orders) have an effect on the surface temperature (see Fig. 1 for an example from a previous work focused on Vesta [1]), and both long-wavelength (thermal) and short-wavelength (visible) radiation must be treated. In "high visibility" environments, such as the interior of a crater, the computational cost of large thermal models is dominated by element-to-element flux calculations, and typically have $O(N^2)$ cost, where N is the number of elements used in the discretization. In recent work [1, 2, 3], we presented preliminary results that used the radiosity method (a boundary element method developed in the mechanical engineering and computer graphics communities) for large-scale thermal and Lambertian illumination modeling on small bodies and the Haworth crater at the lunar south pole (Fig. 2). Our current approach applies methods based on algorithms for the fast solution of boundary formulations of elliptic partial differential equations to allow us to solve the radiosity system of equations in $O(N \log N)$ time. The initial $O(N^3)$ construction cost (Fig. 3) is quickly amortized given the large number of time steps necessary for an accurate thermal simulation.

Outline: Focusing on the lunar south pole, we will:

- present scaled up models which take full advantage of the available high-resolution DEMs [4] of the lunar south pole,
- present a numerical study using a multiresolution hierarchy of DEMs that allows us to quickly spin up a thermal model (1D vertically with multiple depth levels [5]) before time-stepping at higher resolution during the time interval of interest (see Fig. 4) for an example of the same model used with a high-resolution shape model of Vesta [1]),
- show how far away topography can be incorporated at a coarser resolution using `distmesh` [6],
- and highlight our current Python API.

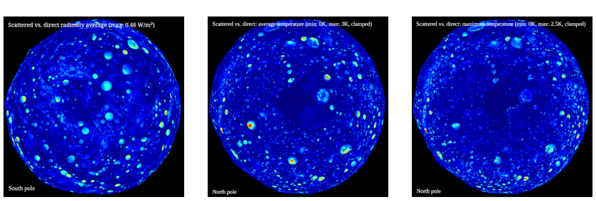


Figure 1: Differences between Vesta flux and temperature maps computed using only direct illumination and higher orders of scattered radiation.

Preliminary results: In previous work, we presented preliminary models for small bodies (Vesta and Ceres) based on a straightforward sparse matrix storage of the radiosity system of equations and solution using a Gauss-Seidel iteration, with models containing up to 800,000 elements. In this case, since most elements are pairwise occluded, a sparse matrix approach allowed us to reach relatively large model sizes. More recently, we presented a proof of concept of the current approach, solving small systems containing at most 25,000 elements in a dense and highly visible environment (Haworth), demonstrating $O(N \log N)$ scaling (Fig. 3).

Our solver works by using a spatial data structure (e.g., a quadtree or octree) to partition the shape model, thereby providing a block structuring of the view factor matrix. Under this permutation, the off-diagonal blocks are low-rank and can be stored and multiplied with $O(N)$ time and space requirements using an SVD. Since the radiosity system can be solved in a small (3–6) number of iterations, an $O(N \log N)$ multiply gives an $O(N \log N)$ solve (this is what it means to be a "direct solver").We plan to demonstrate that this approach can scale to a large number of elements (i.e., greater than 1M elements) at the meeting.

References

[1] Samuel Francis Potter and Erwan M Mazarico. Illumination and thermal environment at (4) Vesta and (1) Ceres. In *AGU Fall Meeting Abstracts*. 2018.

[2] E Mazarico, et al. Illumination and thermal conditions at small bodies. In *Lunar and Planetary Science Conference*, volume 50. 2019.

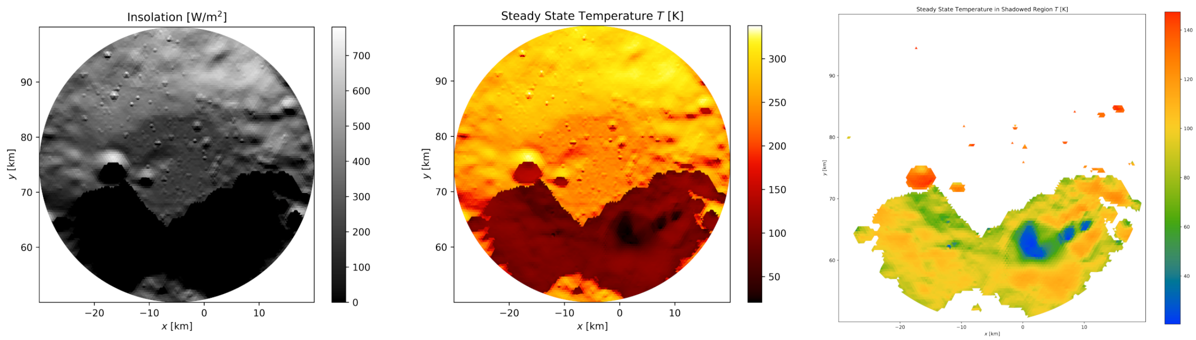
[3] Samuel Francis Potter, et al. Illumination and temperature on rough terrain: Fast methods for solving the radiosity equation. In *AGU Fall Meeting 2019*. AGU, 2019.

[4] David E Smith, et al. Summary of the results from the lunar orbiter laser altimeter after seven years in lunar orbit. *Icarus*, 283:70–91, 2017.

[5] N Schörghofer. Planetary-Code-Collection: Thermal and ice evolution models for planetary surfaces v1.1.4, 2017.

[6] Per-Olof Persson and Gilbert Strang. A simple mesh generator in matlab. *SIAM review*, 46(2):329–345, 2004.

[7] Andrew P Ingersoll, et al. Stability of polar frosts in spherical bowl-shaped craters on the moon, mercury, and mars. *Icarus*, 100(1):40–47, 1992.

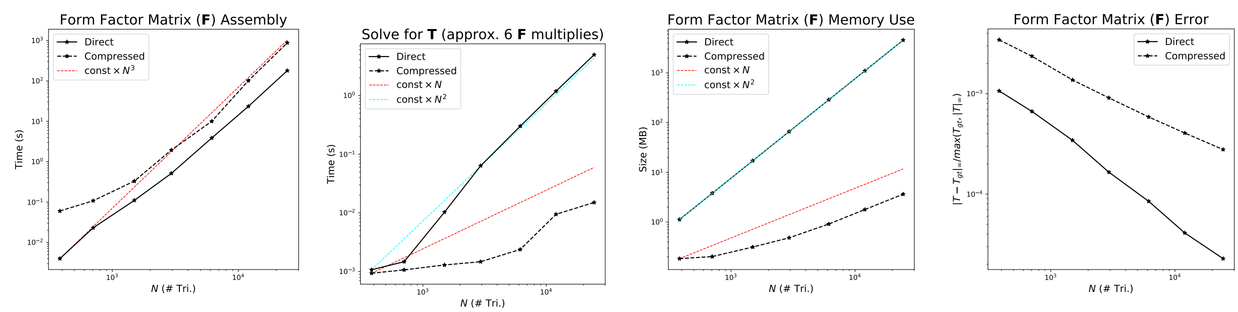


(a) Direct insolation of Haworth crater calculated with sun low on horizon.

(b) Equilibrium temperature of Haworth crater.

(c) Steady state temperature in shadowed region.

Figure 2: Numerical studies steady state temperature of Haworth crater using accelerated solver. We note that in this figure, the rim of the crater was not included in the model. For our results in the meeting, we plan on incorporating far off topography using level-of-detail refinment.



(a) Time to assemble compressed/uncompressed view factor matrices.

(b) Multiplication time with each type of view factor matrix.

(c) Memory use for compressed/uncompressed view factor matrices.

(d) Numerical error using groundtruth steady state temperature in shadow.

Figure 3: Numerical studies for accelerated solver using a problem with a groundtruth solution, the bowl-shaped crater [7].

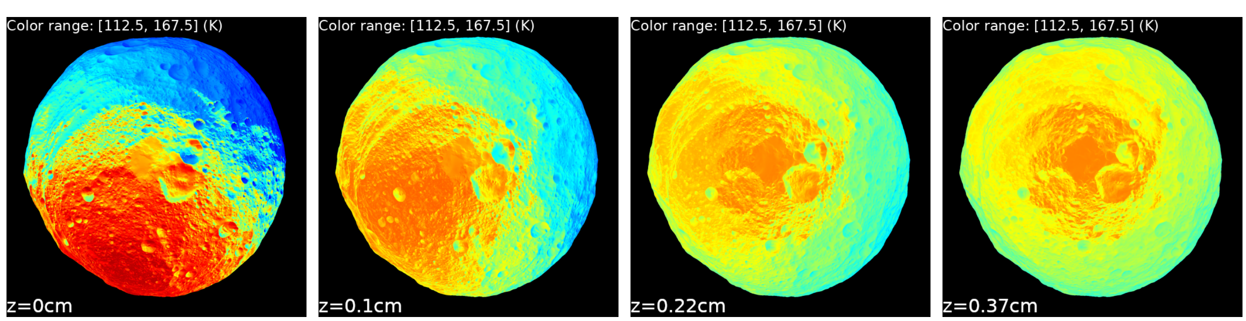


Figure 4: Time-dependent temperature at different z-layers computed using 1D thermal model for subsurface conduction at each element in the discretization. The thermal models were spun up by first running models on decimated meshes and then transferring solutions progressively to finer meshes to allow for a period of burn in.