EQUILIBRIUM TEMPERATURES AND DIRECTIONAL EMISSIVITY OF SUNLIT ROUGH SURFACES WITH APPLICATIONS TO THE MOON L. Rubanenko[1], N. Schorghofer[2], B. T. Greenhagen[3], D. A. Paige[1] [1]University of California, Los Angeles, CA, USA [2]Planetary Science Institute, Tucson, AZ, USA [3]Applied Physics Laboratory, Johns Hopkins University, MD, USA (liorr@ucla.edu)

Introduction: solar radiation dominates the heat balance incident on the surface of airless planetary bodies. Surface roughness affects the equilibrium temperature distribution by changing the solar incidence angle local to each slope. In order to compute surface temperatures and directional emissivity from rough topographies, thermophysical models usually employ computationally expensive techniques such as ray-tracing [1, 2, 3, 4, 5, 6]. Here we assume Lambert scattering to derive closed form expressions for the incident flux and equilibrium surface temperature distribution of sunlit rough surfaces, assuming their slope distribution is Gaussian. Using these distributions, we derive closed-form expressions for the surface infrared brightness as a function of emission angle when the Sun is near the zenith and use them to compute the roughness of the lunar surface on lateral scales of ~ 1 cm. Our methods may be expanded for unresolved, airless planetary bodies in approximate thermal equilibrium such as low thermal inertia, slowly rotating asteroids.

Theory: A Gaussian random surface is a common statistical model for rough topographies that assumes the components of the gradient vector, p, q, are normally distributed with zero mean and root mean square ω [7, 8, 1]. The slope angle may be calculated as $\tan\alpha = \sqrt{p^2+q^2}$, and the slope aspect (compass direction) is $\tan\theta = q/p$. Employing change of variables [9], we may compute the probability density function of the surface slope angles,

$$f_\alpha(\alpha) = \frac{\tan\alpha}{\omega^2\cos^2\alpha}\exp\left(-\frac{\tan^2\alpha}{2\omega^2}\right) \tag{1}$$

and the probability density function of slope aspects,

$$f_\theta(\theta) = \frac{1}{2\pi}. \tag{2}$$

The local solar incidence angle, defined as the angle between the Sun and the slope normal vector, is given by,

$$\cos\Theta = \cos z\cos\alpha + \sin z\sin\alpha\cos(\theta - a_s). \tag{3}$$

where z is the solar zenith angle, a_s is the solar azimuth angle and θ is the slope aspect. The flux reaching each slope on the surface depends on the cosine of the incidence angle and the distance to the sun r,

$$F = \frac{S_0\,(1-A)}{(r/1\,\mathrm{AU})^2}\cos\Theta \equiv \beta\cos\Theta \tag{4}$$

where $S_0 = 1367\ \mathrm{Wm^{-2}}$ is the average solar constant at $1\mathrm{AU}$ and A is the surface albedo. Finally, the equilibrium temperature may be derived from the incident flux assuming the surface is a black-body with emissivity ε. Throughout this work, we assume radiative equilibrium, $A = 0.1$ and $\varepsilon = 0.95$ as example values for the bond albedo and emissivity of the lunar surface.

Results: *Temperatures and Directional Emissivity at local noon:* At the local noon, the solar zenith angle z is small and

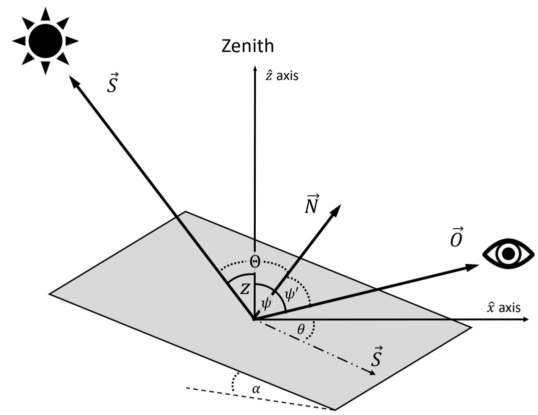

Figure 1: The angles and vectors defining the surface slope distribution and illumination conditions. $\vec{N}$ is the surface normal, α the slope angle, θ the slope aspect and z the solar zenith angle.

Eq. 3 is the same as Eq. 1 with $\alpha = \Theta$. In this case, it is straightforward to obtain closed form expressions for the incident solar flux distribution,

$$f_{F_0}(F) = \frac{\beta^2}{\omega^2 F^3}\exp\left(-\frac{1}{2\omega^2}\frac{\beta^2 - F^2}{F^2}\right), \tag{5}$$

and the equilibrium temperature distribution,

$$f_{T_0}(T) = \frac{4}{\omega^2\rho^2 T^9}\exp\left(-\frac{1}{2\omega^2}\frac{1-\rho^2 T^8}{\rho^2 T^8}\right). \tag{6}$$

where we defined $\rho \equiv \sigma\varepsilon/\beta$. By integrating Eq. 6, we may obtain a useful closed form expression for the mean equilibrium temperature of sunlit rough Gaussian surfaces,

$$\bar{T_0} = \frac{1}{(2\omega^2\rho^2)^{1/8}}\Gamma\left(\frac{7}{8}, \frac{1}{2\omega^2}\right)\exp\left(\frac{1}{2\omega^2}\right) \tag{7}$$

In Figure 3 we show the mean equilibrium surface temperature computed by our analytic model relative to a numerical model that includes ray casting. As expected, our model becomes less accurate when the Sun is not near the zenith. However, for low-moderate zenith angles and surface roughness values our model error is contained under 2%. We now use these distributions to calculate the mean infrared brightness as a function of emission angle $\bar{B}(\psi)$ by averaging the total energy reaching the observer from all surface slopes, normalized and divided by the projected area,

$$\bar{B}(\psi) = \frac{\int B\cos\psi' dA}{\int \cos\psi' dA}. \tag{8}$$

where ψ' is the angle between the slope normal vector and the observer. For a Gaussian surface illuminated from zenith we find a closed-form solution for this integral,

$$\bar{B}(\psi) = \bar{B}(0) - \frac{\bar{B}(0) - \bar{B}\left(\frac{\pi}{2}\right)}{\bar{B}\left(\frac{\pi}{2}\right)} \tilde{B}(\psi), \tag{9}$$

where,

$$\begin{aligned}\tilde{B}(\psi) =& \frac{\bar{B}(0)}{2\pi I \omega^2} \frac{1}{1+\Lambda(\cot\psi)} \cdot \\ & \frac{\tan\psi + \cot\psi}{2} \exp\left(\frac{1-\cot^2}{4\omega^2}\right) \cdot \\ & \left[K_0\left(\frac{1+\cot^2}{4\omega^2}\right) - K_1\left(\frac{1+\cot^2}{4\omega^2}\right)\right].\end{aligned} \tag{10}$$

and,

$$\begin{aligned}\tilde{B}\left(\frac{\pi}{2}\right) =& \sqrt{\frac{2\pi}{\omega^2}} \frac{\bar{B}(0)}{4\pi I \omega^2} \exp\left(\frac{1}{4\omega^2}\right) \\ & \left[K_1\left(\frac{1}{4\omega^2}\right) - K_0\left(\frac{1}{4\omega^2}\right)\right]\end{aligned} \tag{11}$$

and where $K_\nu(x)$ is the ν'th order Modified Bessel Function of the Second Kind and Λ is the shadowing function due to Smith [8]. In Figure 2 we show two sets of measurements of lunar infrared brightness obtained through telescopic observations [10] and Diviner measurements [5]. Using non-linear regression we fit our analytic model to these directional infrared brightness measurements to find the root mean square slope ω. Our measurements agree well with previous measurements at the $0.1 - 1$ mm lateral scale, [11].

Temperature Distribution for any Solar Zenith Angle: In the general case, Eq. 3 can no longer be reduced as in the special $z = 0$ case and deriving the incidence angle, flux and temperature distribution requires solving an integral with no-closed form solution,

$$I = \int_a^b \frac{\left[(x-a)(b-x)\right]^{-\frac{1}{2}}}{x^3} e^{-\frac{1}{2\omega^2 x^2}} dx \tag{12}$$

where $a = \cos(\Theta - z)$ and $b = \cos(\Theta + z)$. Here we obtain an asymptotic approximation to the integral using Laplace's method [12] which is accurate for small ω^2 (low roughness) and $\Theta \not\approx z$ (sun-facing surface slopes). Using this approximate closed-form solution and change of variables, we obtain the equilibrium temperature distribution of sunlit Gaussian surfaces for any solar zenith angles,

$$\begin{aligned}f_T(T) \approx& \frac{4\omega\rho T^3}{\sqrt{2\pi}} \frac{\sqrt{1+\tau_2\cot z}}{\tau_1} \\ & \left(1+\frac{1}{\omega^2\tau_3}\right) \exp\left[\frac{1}{2\omega^2}\left(1-\frac{1}{\tau_1^2\tau_3}\right)\right],\end{aligned} \tag{13}$$

where

$$\begin{aligned}\tau_1 &= \sqrt{1-\rho^2 T^8} \\ \tau_2 &= \frac{\rho T^4 \cot z}{\tau_1} \\ \tau_3 &= \tau_1^2 (1+\tau_2)^2 \sin^2 z.\end{aligned} \tag{14}$$

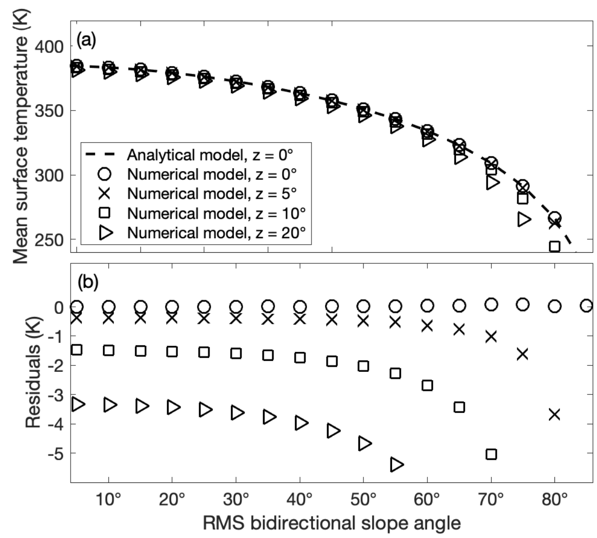

Figure 2: (a) Mean surface temperatures computed by our analytic model compared to the mean surface temperature computed by a numerical model. (b) Residual plot for (a).

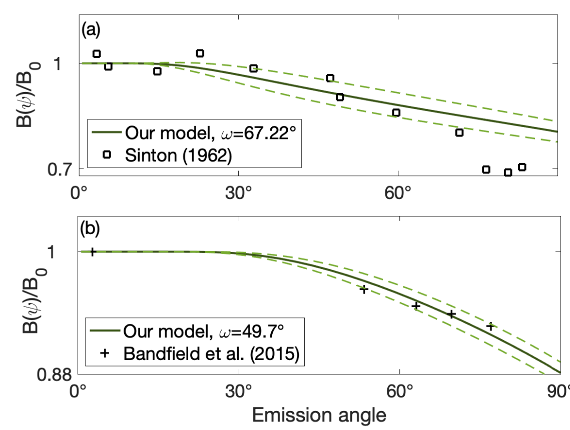

Figure 3: We use our model to invert for the bidirectional root mean square slope (magnitude of surface roughness) at the lateral scale that affects the thermal phase function. (a) Telescopic observations. (b) Diviner measurements.

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