

Scaling Laws for the Geometry of an Impact-Induced Magma Ocean. Miki Nakajima¹, Gregor J. Golabek², Kai Wünnemann³, David C. Rubie², Christoph Burger⁴, Lukas Manske³, Henry J. Melosh⁵, Seth A. Jacobson⁶, Francis Nimmo⁷, and Scott D. Hull¹, ¹University of Rochester, 227 Hutchison Hall, Rochester, NY 14627, USA (mnakajima@rochester.edu). ²University of Bayreuth, Universitätsstrasse 30, 95440 Bayreuth, Germany. ³Museum für Naturkunde Berlin, Invalidenstrasse 43, 10115 Berlin, Germany. ⁴University of Tübingen, Auf der Morgenstelle 10, 72076 Tübingen, Germany. ⁵Purdue University, 550 Stadium Mall Drive, West Lafayette, IN 47907, USA. ⁶Michigan State University, 288 Farm Lane, East Lansing, MI 48823, USA. ⁷University of California Santa Cruz, 1156 High Street, Santa Cruz, CA 95064, USA.

Summary:

We developed scaling laws for the mass fractions of the impact-induced magma oceans and their geometry based on more than 100 smoothed particle hydrodynamic (SPH) simulations. The input parameters include the impact velocity, impactor-to-total mass ratio, impact angle, and total mass. We find that the total internal energy increase by a large impact and the geometry of a magma ocean are well represented by the Legendre polynomials. We find that the equilibrium pressure at the base of a melt pool model is generally higher (10-50%) than those at the base of a radially uniform magma ocean model. This could have a significant effect on understanding the metal-silicate equilibration process during planet formation and therefore the chemical evolution of the planetary core and mantle.

Introduction:

The compositions of planetary cores and mantles evolve over time by impact processes. An impact melts the outer part of the planetary mantle and forms a magma ocean, and the impactor's iron experiences metal-silicate equilibration with the ambient silicate liquid. This process determines the partitioning of elements between iron and silicate and therefore the evolving chemical composition of both mantle and core of the planet. The equilibration process is controlled by parameters including the equilibrium pressures P_{eq} and temperatures T_{eq} [e.g., 1].

Conventionally, the values of T_{eq} and P_{eq} are often associated with or assumed to be proportional to the values at the bottom of a global magma ocean (Figure 1a) [e.g., 2]. However, an impact can produce a spatially confined melt pool (Figure 1b) that centers around the impact point. This would provide higher T_{eq} and P_{eq} at its bottom while having the same melt volume. Due to isostatic adjustment this melt pool would radially spread out and become a global magma ocean over time, but this timescale (a rough estimate is tens of years [e.g. 3]) is likely to be much longer than the equilibration timescale, ranging from hours to months. Therefore, a melt pool is likely to be the more relevant geometry of a magma ocean when considering the equilibrium pressures and temperatures.

There are a number of insightful studies on scaling laws for the impact-induced melt volume [e.g., 4, 5], however, there is no study that predicts the geometry of a magma ocean. Such scaling laws would be useful for the community to predict the equilibration conditions and, thus, resulting planetary chemical compositions. For this purpose, we present scaling laws that describe the geometry of an impact-induced magma ocean for the first time based on more than 100 giant impact simulations.

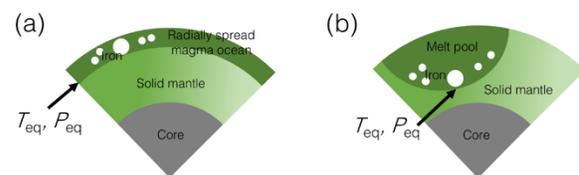


Figure 1: Schematic views of (a) a global magma ocean model, and (b) a spatially confined melt pool model.

Model:

We perform giant impact simulations using the SPH method (Figure 2a). The input parameters are the impact angle θ (0° , 30° , 60° , and 90°), total mass M_T ($1M_{Mars}$ - $53M_{Mars}$, where M_{Mars} is the Mars mass), impact velocity v_{imp} (v_{esc} - $2v_{esc}$, where v_{esc} is the mutual escape velocity), and impactor-to-total mass ratio γ (0.1-0.5). We used the M-ANEOS equation of state to describe both mantle and core. We modeled the extent of mantle heating through two stages: (1) quantification of the total (bulk) internal energy gain by impact, and (2) determination of the heat distribution within the mantle.

Results and discussion:

We find that the total internal energy gain is well modeled as

$$\Delta IE(\theta) = (KE_0 + \Delta PE) \sum_{l=0}^{n_c} e_l P_l(\cos \theta) \quad (1)$$

where KE_0 is the initial kinetic energy and ΔPE is the potential energy change due to the impact. These two parameters are functions of M_T , v_{imp} , and γ . Here, P_l are the Legendre polynomials and e_l are coefficients. n_c is chosen to be 6 in our model.

Our numerical simulations show that the total internal energy gain ΔIE due to an impact is well

characterized by the Legendre polynomials especially at relatively small impact velocities ($v_{\text{imp}} < 1.4-1.5 v_{\text{esc}}$). Our bulk heating model generally agrees with previous studies on melt volume estimates [e.g., 5], whereas it predicts higher internal heating even when the impact angle is 90° (the most grazing impact). The difference originates from our model taking into account tidal deformation and heating whereas previous models do not consider this effect.

Secondly, the heat distribution within the mantle is modeled as

$$\Delta U(r', \psi, \theta) = F(r', \psi) \Delta IE(\theta) \quad (2)$$

where

$$F(r', \psi) = \sum_{m=-2}^2 \sum_{l=0}^2 c_{l+3(m+2)} r'^{lm} P_l(\cos \psi) \quad (3)$$

where F describes the heat distribution as a function of r' and ψ , where r' is the radial distance normalized by the planetary radius and ψ is the colatitude within the mantle from the most heated region which usually coincides with the impact point (Figure 2a). $c_{l+3(m+2)}$ are the coefficients for the Legendre polynomials. The coefficients including e and c are determined by minimizing the error between the SPH simulations and our model.

An example of our SPH simulation and heat distribution model are shown in Figure 2b and 2c, respectively, where $\theta = 60^\circ$ and $v_{\text{imp}} = v_{\text{esc}}$. The color contour shows the internal energy gain due to the impact. Our model generally captures the geometry of the impact-induced heated region, whereas the localized heating due to accretion of the impactor, which can be seen around $0.6 < r' < 0.7$ and $-100^\circ < \psi < 30^\circ$ (Figure 2b), is not well captured in our model (Figure 2c). Overall, our heat distribution model works well when the impact angles are 0° , 60° , and 90° , whereas it does not capture the SPH outcome well when the impact angle is 30° . This is because an impact at this angle tends to heat the mantle asymmetrically, while our model assumes symmetry.

The shape of a magma ocean can be determined by combining our model with a melting criterion. For example, a melt region can be defined as a region where the internal energy exceeds the latent heat for the mantle melting. When the latent heat is $8.3 \times 10^5 \text{ J/kg}$, our calculations show that the magma pool model (Figure 1b) predicts 10-50 % larger pressures than the conventional global magma ocean does (Figure 1a). This implies that impactor's iron would experience metal-silicate equilibration at higher pressures than previously expected, which can significantly affect the resulting chemical compositions of both core and mantle of a planet, and therefore important parameters including the planet's

volatile content and the concentrations of light elements of the core.

References:

- [1] Li, J., and Agee, C. B., (1996). *Nature*, 381, 686. [2] Rubie, D. C., et al, (2015). *Icarus*, 248, 89. [3] Reese, C. C., and Solomatov, V. S., (2006). *Icarus*, 184, 102. [4] Pierazzo, E., and Melosh, H. J. (2000). *Icarus*, 145, 252. [5] Abramov, O., et al., (2012). *Icarus*, 218, 906.

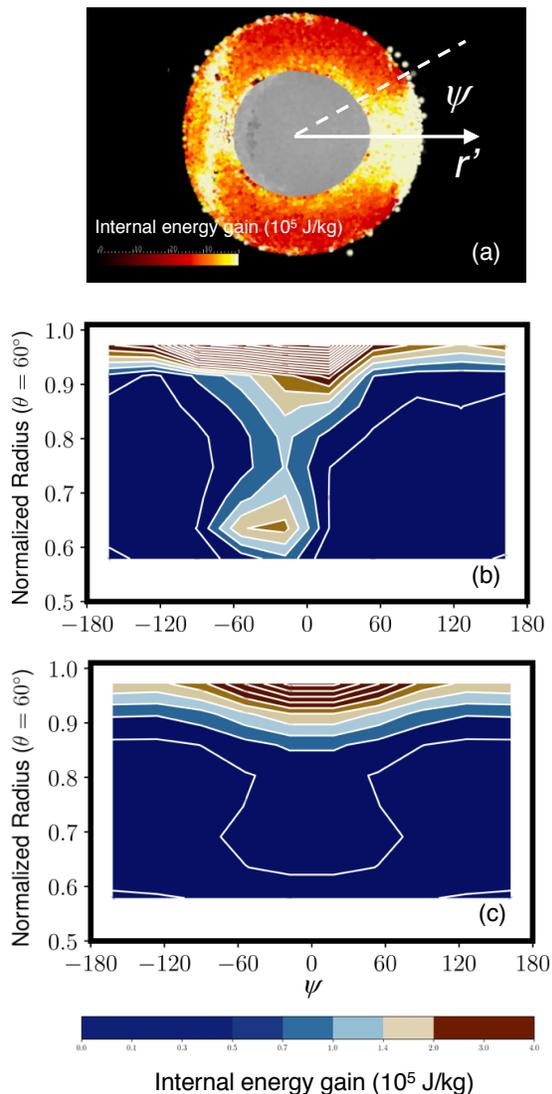


Figure 2: (a) An example of a SPH simulation ($M_T = M_{\text{Mars}}$, $v_{\text{imp}} = v_{\text{esc}}$, $\gamma = 0.1$, $\theta = 0^\circ$) with schematic descriptions of the normalized radius r' and the angle ψ , which is the angle counting from the most heated region of the mantle. The color shows the internal energy of the mantle in 10^5 J/kg . (b) Contour of the internal energy gain based on a SPH impact simulation. (c) Same as (b) with our heating model. In (b) and (c), the parameters used are $M_T = M_{\text{Mars}}$, $v_{\text{imp}} = v_{\text{esc}}$, $\gamma = 0.1$, and $\theta = 60^\circ$.