STRESS-DEPENDENT TIDAL DISSIPATION IN ICE AND OLIVINE. H. J. Melosh[1] D. L. Goldsby[2] and L.N. Hansen[3], [1]EAPS Department, Purdue University, West Lafayette, IN 47907, jmelosh@purdue.edu, [2]EES Dept, U. of Pennsylvania, Philadelphia PA, 19104, [3]Dept. Earth Sciences, U. of Minnesota, Minneapolis, MN 55455

Introduction: Most previous work on creep deformation of rocks and ice focused on long-term steady-state creep. While this is the right goal for understanding long duration tectonic processes, such as those that underlie plate tectonics on Earth, tidal stresses apply periodic reversals in stress and strain on timescales comparable to orbital periods. In our solar system these periods range from days (solar and lunar tides on Earth) to a month (tides in our Moon) and periods in between (the Galilean satellites of Jupiter). Exoplanet tidal periods are comparable to these. In the past, some estimates of tidal dissipation estimates have used steady-state creep rheologies. However, it has been known from the days of Griggs's experiments [1] that steady state creep is preceded by a short period of "transient" or "primary" creep that involves much shorter time scales and more rapid flow. This means that dissipation is much larger at tidal timescales than that implied by steady-state rheology. McCarthy et al. [2] and McCarthy et al [3] have partly remedied this situation for water ice under conditions of high tidal stresses, but little work has so far been done in this regime for minerals, such as olivine, that comprise rocky planet interiors.

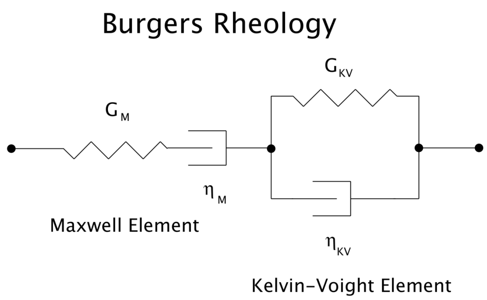

Figure 1: Schematic illustration of the Burgers rheology, which requires the specification of two elastic shear moduli and two viscosities. Unlike most treatments of this rheology, laboratory data indicates that one or both viscosities depend on the shear stress raised to a power greater than one.

The rheology of solid earth materials is often represented by idealized flow laws, of which the simplest are Hooke's law of elastic deformation, which links stress and strain in a linear fashion, and Newton's viscous law that relates stress and strain rate. More complex behavior can be compounded of these simple elements. A Maxwell model that links elastic and viscous flow in series is often used to represent long term creep in Earth's mantle, and does a good job of representing this behavior, although at high stresses it must be supplemented by a viscous element in which strain rate is a power law function of stress, a circumstance that adds some interesting complications to linear Newtonian viscosity [4]. However, the simplest model that can represent both transient creep and steady state creep is a so-called Burger's solid (Figure 1) that incorporates both a Maxwell element and the related Kelvin-Voight element that puts an elastic element and a viscous element in parallel. In the present analysis we assume that both viscous elements may be functions of the applied stress, a complexity suggested by experimental data.

Method We used such a Burger's model to incorporate pilot experiments on ice and olivine. The nonlinear differential equations describing the system were derived, checked and numerically integrated with the help of *Mathematica©*. The behavior of this four-component system is already quite interesting. Because the flow laws at high applied stresses are nonlinear functions of stress, the dissipation under periodic loads is amplitude dependent. This is not the case for linear viscous rheologies: In this case the dissipation factor Q^{-1} would be independent of amplitude, but for the observed power laws Q^{-1} increases rapidly with the amplitude of the periodic stress. The behavior of the system also depends on the period of forcing and, for the sake of illustration, we assume a period of 10^5 seconds (about 1.2 days), similar to that of many exoplanets discovered around low-mass stars. Again, for definiteness, we assume a sinusoidal variation in the stress, although we are well aware that the tidal forcing by an orbiting planet, while periodic, is not sinusoidal because of the combined timing variation from Kepler's second law and the $1/a^3$ dependence of tidal stress. Because these variations cannot be expressed as analytic functions, we reserve for the future numerical application of the flow laws to actual systems.

Results: Figure 2 illustrates the dependence of Q^{-1} on the amplitude of the stress variation. Our data for olivine at a temperature of 1473 K are summarized in the table. We similarly show preliminary data for ice, also summarized in the table, which we have evaluated at the low temperature of 183 K (the same homologous temperature as the olivine in order to keep both curves on the same plot). The viscosity dependence on stress is derived from preliminary observations of both transient creep and steady state creep in both ice and olivine. We also need estimates for the two elastic shear moduli in the Maxwell and Kelvin-Voight elements. The Maxwell elasticity is derived from high-frequency elastic response from either laboratory ultrasonic measurements or seismic data. The Kelvin-Voight elasticity can, in principle, be derived from very slow elastic response. It is often a rule of thumb in geodynamics to presume that the long-term elastic modulus is about half of the modulus derived from seismic data, which implies that the Kelvin-Voight elasticity is equal to the Maxwell elasticity, and we adopt that ansatz for the current illustrative computations in Figure 2, although this should be subject to future observational study. Note that the amplitudes of the assumed tidal stress variation are much larger than typical plate tectonic stresses, which average only a few MPa. This stress is far larger than that which acts on the Earth or Moon today, although it could well have been important for the early Moon during a proposed era of evolution through the evection resonance [5]. Because such tidal stresses far exceed the background tectonic stress, the apparent rheological linearization that occurs for small stress fluctuations imposed on a larger stress background [4] does not occur and the inherent nonlinear amplitude dependence of the flow will dominate the dissipation.

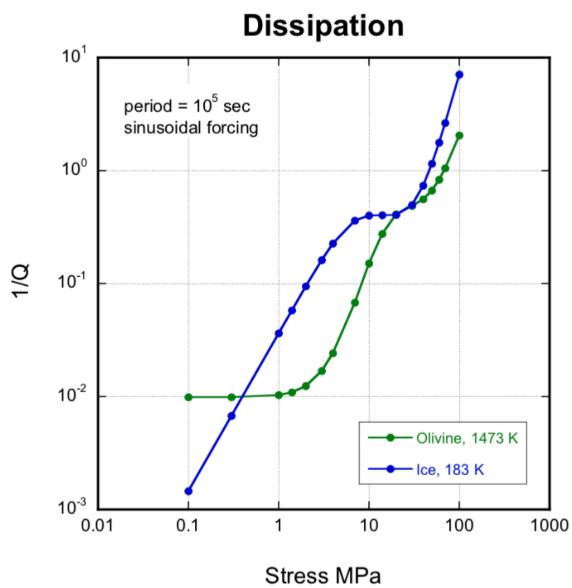

Figure 2: Dissipation as a function of stress amplitude for olivine at 1473 K and ice at 183 K (similar homologous temperatures) using preliminary data from our pilot study. The prominent swoop at around 10 MPa in both materials corresponds to a change in dissipation from the Kelvin-Voight (transient creep) element at low stresses to the Maxwell (steady-state creep) element at high stress.

Figure 2 is based on the numerical integration of the nonlinear equations describing a Burgers rheological model. This has already been implemented in a Mathematica notebook and checked rigorously. At low stresses the olivine curve is flat—that is, Q^{-1} does not depend on amplitude, reflecting a presumed mechanism of Newtonian grain-boundary sliding. The ice curve is, however, derived from a grain boundary sliding flow law that does depend on the stress amplitude, although not as strongly as the steady state creep law. At stresses above about 1 MPa the dissipation in olivine becomes stress-dependent. The most prominent feature of both curves is the sudden bend at about 10 MPa. This bend reflects a change in the dissipation mechanism: At low stresses, most dissipation occurs by transient creep in the Kelvin-Voight element. However, at high stresses this low-viscosity element relaxes completely and the dissipation is then dominantly in the Maxwell element, accounting for the deflection of both curves toward lower dissipation.

Table: Pilot study rheological parameters of ice and olivine*

Property	Olivine, 1473 K	Ice, 183 K
Maxwell shear modulus, G_M, Pa	7.5×10^{10}	3.46×10^9
Kelvin-Voight shear modulus, G_{KV}, Pa	7.5×10^{10}	3.46×10^9
Maxwell viscosity, η_M, Pa-s	1.8×10^{18}, $\sigma < 1$ MPa $2.84 \times 10^{34}\, \sigma^{-2.5}$, $\sigma > 1$ MPa	$3.01 \times 10^{36}\, \sigma^{-3}$
Kelvin-Voight viscosity, η_{KV}, Pa-s	1.35×10^{17}, $\sigma < 1$ MPa $2.10 \times 10^{33}\, \sigma^{-2.5}$, $\sigma > 1$ MPa	$3.05 \times 10^{23}\, \sigma^{-1.4}$

*σ is the second invariant of the deviatoric stress tensor, units are Pa unless otherwise noted

Conclusions: Stress nonlinearity implies that, depending on the stress amplitude, the dominant process in tidal dissipation can switch from transient creep to steady-state creep. In both ice and olivine, high stress amplitude de-emphasizes transient creep and dissipation may be dominated by steady-state flow at much lower dissipation rates. These results indicate the importance of characterizing both transient and steady-state creep to properly define tidal energy dissipation in planets.

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