CALCULATING GRAVITY ACCELERATION SPECTRA FROM INDIVIDUAL SPACECRAFT ARCS: PROOF OF CONCEPT WITH MESSENGER DATA. P. B. James[1], [1] Baylor University, Department of Geosciences, One Bear Place #97354, Waco, TX 76798, USA (email: P_James@baylor.edu).

Introduction: The extended mission of the MESSENGER spacecraft incorporated a series of low-altitude arcs [1]. The line-of-sight (LOS) accelerations from a single arc can provide information about the power spectrum of the gravity field. From this spectrum, we can estimate quantities such as the radius of the body's rocky surface and the RMS amplitude of topography. This technique will be particularly valuable for future fly-by tours of ocean worlds in the outer Solar System, at which the rocky surfaces are hidden under ice and water. In this work, we demonstrate the technique for extracting spectral information out of one particular arc, #1273 of the MESSENGER mission.

Amplitude spectra from Fourier transforms: Generally speaking, the Fourier transform of an acceleration a along a ground-track with length T is:

$$\hat{a}(k) = \frac{1}{T}\int_{-T/2}^{T/2} a(x)e^{\frac{2\pi ikx}{T}}dx$$

where the wavenumber k is related to spherical harmonic degree l through Jeans' relation:

$$k = \sqrt{l(l+1)}\frac{T}{2\pi R}$$

where R is the interface radius. The amplitude spectrum is simply the magnitude of the wavenumber-domain accelerations $\hat{a}(k)$.

The 1-D multi-taper method: We wish to localize LOS accelerations on an orbital track for a couple of reasons. In particular, the altitude of the spacecraft changes throughout an arc, so we want to be able to separately interpret independent spectra from different segments of an arc. Furthermore, multitaper estimation with several independent spectra allows us to reduce the resulting spectral variance.

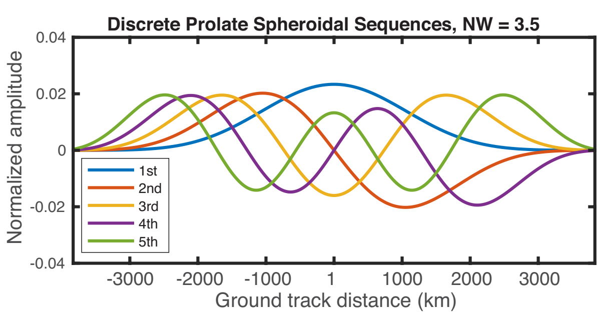

Figure 1: Five DPSS tapers for our data

In order to minimize spectral leakage, we employ discrete prolate spheroidal sequences (DPSS) with a bandwidth of 2NW = 7. DPSS tapers are functions designed to maximize the energy of a time series within a specified ground-track while simultaneously minimizing spectral leakage [2]. The first five such tapers (which localize >99% of the signal energy within the track) are plotted in Figure 1. These tapers can be multiplied by the LOS accelerations $a(x)$ and transformed to produce a localized amplitude spectrum.

Expected LOS acceleration spectra: We can calculate the acceleration spectrum that would arise from a spacecraft traversing through a gravity field that globally obeys a power law known as the "Kaula rule" [3]: $S_{CC} \approx Al^{-2}$, where S_{CC} is the power of the dimensionless gravity coefficients, l is spherical harmonic degree, and A is an empirical constant. However, it is not trivial to translate a Kaula rule into a LOS spectrum: the signal associated with an effective spherical harmonic degree l is influenced by the global gravity field at the degree l, but also from global gravity coefficients at higher degrees. Taking this into account, the expected amplitude spectrum for a wavenumber k is:

$$E[|\hat{a}(k)|] = \frac{GM}{r^2}A\sqrt{2\sum_{l=m}^{\infty}\left(\bar{P}_{lm}(0)\left(\frac{R_0}{r}\right)^l\frac{l+1}{l^2}\right)^2}$$

where E is "expectation". The observed and predicted amplitude spectrum for the first DPSS taper is plotted in Fig. 2, along with contribution of instrument noise.

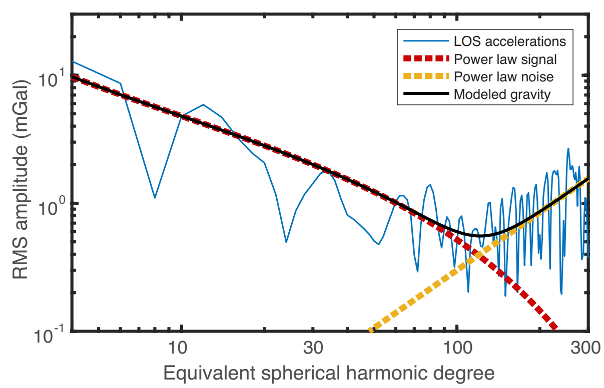

Figure 2: An example of an observed acceleration spectrum (blue) corresponding to Arc 1273 of the MESSENGER spacecraft localized with the first taper plotted in Figure 1. The red dashed line indicates the expected amplitude of the LOS accelerations produced by a gravity field that follows Kaula's law. The misfit between the observed and modelled gravity is plotted in Figure 3a for various values of the rocky surface radius R_0 and the Kaula coefficient A.

Using spectra to estimate the radius of a body's rocky surface: For most bodies in the Solar System, the surface of the body's outermost rocky layer is the most significant contributor to the body's gravity field. Consequently, the spectrum may be used to estimate the radius of the body. Since the expected amplitude of the LOS spectrum is also dependent on the Kaula rule—which is in turn dependent on the amplitude of surface topography—we can also use the power spectrum to determine this amplitude.

We explore this technique for Arc 1273 of the MESSENGER spacecraft. The first step of this process is to interpolate the spacecraft accelerations to a ground track with regular along-track spacings of 1 km. This yields a timeseries of LOS accelerations, $a(x)$, taking into account the geometry of the LOS vector. We calculate misfit between observed and modelled spectra by taking the square root of the mean of the squares of the misfits at each wavenumber. This RMS misfit value is calculated for a variety of rocky surface radii R_0 and a variety of Kaula rule coefficients A, as shown in Figure 3.

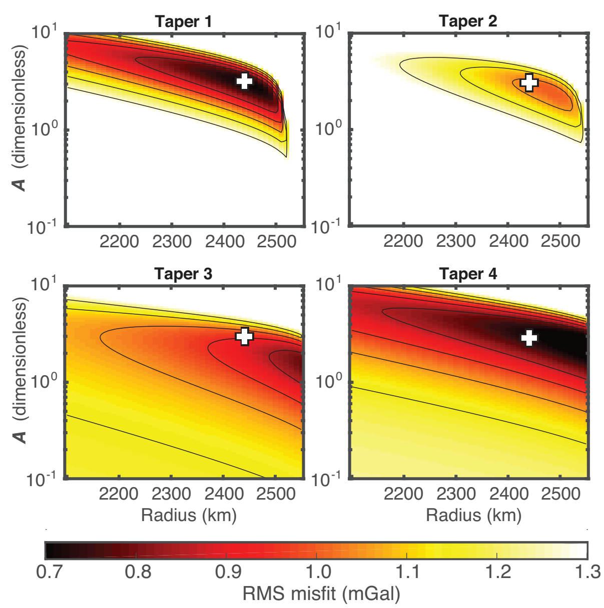

Figure 3: The RMS misfit between the observed acceleration spectrum and the modeled amplitude, using the first four DPSS tapers plotted in Fig. 1. <u>The true values of Mercury's radius and gravity power law coefficient are indicated by the white crosses.</u>

The RMS misfits plotted in Fig. 3 correspond to the DPSS tapers shown in Fig. 1, and the minima of these plots represent independent estimates of R_0 and A. Using the multitaper method, we can combine these into a single misfit plot, shown in Fig. 4. The minimum misfits can then be compared to the true values of Mercury's radius and Kaula rule (indicated by the white crosses in Figures 3 & 4).

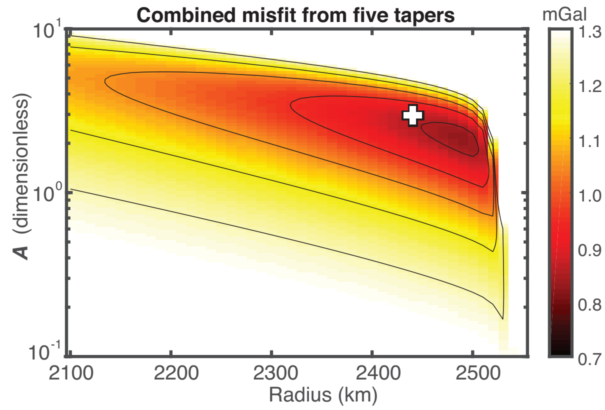

Figure 4: The combined RMS misfit produced by simultaneously modeling the acceleration spectra from the five tapers (four of which are plotted in Fig. 3).

These results demonstrate that the acceleration spectrum of a single track can be used to successfully predict Mercury's radius with an uncertainty of approximately 100 km. The Kaula coefficient A (as well as the RMS amplitude of surface topography) could be successfully recovered within a factor of 2. The incorporation of additional spacecraft arcs is expected to reduce these uncertainties by a factor of $\sqrt{n}$, where n is the number of tracks.

Application to icy bodies: The radius estimation described above is certainly a pointless exercise when it comes to the planet Mercury, since we already know the radius of Mercury to much higher precision through other measurement techniques. However, this method may be the most important tool for estimating the depth of Europa's subsurface ocean. The upcoming Europa Clipper mission will perform a series of flybys of Europa with trajectories and periapsis altitudes similar to those of the MESSENGER extended mission. Europa's rocky ocean floor contributes more the gravity field than the ice shell over a range of wavelengths. Consequently, the method illustrated in Figs. 3 & 4 could be employed to measure the depth of Europa's liquid water ocean.

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References: [1] Leary J. C., et al. (2007) Space Sci. Rev. 131, 187–217. [2] Thomson, D. J. (1982) Proc. of the IEEE 70:9, 1055-1096. [3] Kaula, W. M. (1966) JGR 71:22, 5303-5314.