EXTENDING THE ANALYTIC APPROXIMATION FOR CALCULATING MOMENTUM ENHANCEMENT IN HAZARDOUS OBJECT MITIGATION SCENARIOS WITH A VARIABLE EJECTA VELOCITY. M. L. Harwell\textsuperscript{1}, C. S. Plesk\textsuperscript{1}, S. A. Becker\textsuperscript{1}, C. M. Biwer\textsuperscript{1}, M. B. Boslough\textsuperscript{1}, L. G. Margolin\textsuperscript{1}, \textsuperscript{1}Los Alamos National Laboratory, Los Alamos, NM, USA, (mharwell@lanl.gov).

Introduction: NASA and the NNSA have embarked on a three year inter-agency collaboration to study the possibility of preventing the impact of an asteroid or comet on Earth using three different options: a kinetic impactor, a near-surface nuclear burst, and a stand-off nuclear burst [1]. As the work has progressed, the factor of momentum enhancement, $\beta$, which is the ratio of the change of momentum of the remaining asteroid to that of the impactor, has emerged as a figure of merit between the models [2]. This factor quantifies the efficiency of momentum transferred to the center of mass of the object, and thus the predicted change in velocity, $\Delta v$, from the deflection attempt. However, limitations in how long simulations may progress lead to uncertainty in numerical estimates.


The characteristics of a specific hazardous object will remain mostly unknown. A more complete analytic approximation can provide a useful tool for a sensitivity analysis on variables related to deflection scenarios. This analysis can then inform lengthier and more computationally expensive numerical models.

Analytic methods for calculating $\beta$: The foundational work for calculating the factor of momentum enhancement is produced in Holsapple and Housen, 2012 and provides a lower bound [2].

Classical method: The method introduced builds on pi-scaling by applying a uniform velocity to ejecta from a growing crater. This velocity is the minimum of the escape velocity of the planet, and $v^*$ [2]. The volume of ejecta is approximated as 60\% of the transient crater’s volume. Beta is then computed $\beta = 1 + \left(\frac{p_{\text{projectile}}}{p_{\text{ejecta}}}\right)$, where the ejecta’s momentum, $p_{\text{ejecta}}$, is the product of the minimum velocity and ejecta mass. This method assumes the lowest possible ejection velocity able to satisfy escape criteria for the target object and applies it uniformly to the escaping ejecta mass, providing a low velocity and high mass estimation for ejecta’s momentum.

Updated method: The description for mass of ejecta arises from the streamlines, shown in figure 1, outlined in the Z-model, where the streamlines outline the path followed by the ejecta as it transitions from the contact and compression stage to crater excavation [3]. The equation,

$$M_{\text{tube}}(r) = 2\pi \rho_i \int_{R-\Delta R}^{R} \int_{0}^{\theta} r^2 \sin(\theta) d\theta dr \ \text{for} \ r > R_p$$

approximates the amount of mass excavated within the streamlines [3].

The velocity assigned to each streamline of ejecta emerges from the conservation of energy. The sum of the projectile’s kinetic energy, the gravitational energy of the target body, and energy from the strength of the target in the excavated volume contribute to the ejecta velocity, such that $v_{ej} = \sqrt{\frac{2E_{\text{sum}(r)}}{\rho_i(\Delta r)}}$. A more complete explanation can be found in [2]. This treatment of ejecta constrains the ejecta lofted depending on cratering regime: If $E_{\text{kin}} > E_{\text{gravity}}$ in the gravity cratering regime, then the material is lofted. The same applies in the strength regime, such that if $E_{\text{kin}} > E_{\text{strength}}$ in the strength regime, the material is lofted. A further constraint of ejecta moving at greater than the escape velocity of the object to contribute to the momentum enhancement factor is applied.

This more detailed characterization of ejecta momentum by location within the crater allows for a more physically representative approximation for $\beta$ from crater ejecta.

Figure 1, figure 2a in [5]. Streamlines of excavation flow from forming crater.

Application to analytic approximations: We implemented the updated treatment of ejecta within a python script for pi scaling. To build this in fully, we used shock impedance to find the target’s peak particle shock velocity, which is incorporated in to residual velocity [6, 7]. Our ultimate equation for kinetic energy with residual velocity becomes:

$$E_{\text{kin}}(r) = \pi \rho_i (Cu_{\text{proj}} R_p)^2 \int_{R-\Delta R}^{R} \int_{0}^{\theta} r^2 (1-\text{nm}) \sin(\theta) d\theta dr$$

Note the divergent solution that arises over the integration of the streamline closest to the point of impact. This falls out of the solution when the material excavated within the radius of the projectile is excluded, $[r|r > R_p]$. We then calculated updated momentum for ejecta within each streamline. This material contributed...
to momentum enhancement if the criteria for ejection from the crater under the appropriate cratering regime was met and if the velocity of ejection was greater than the escape velocity of the object.

In this way, we have implemented an analytic approximation for $\beta$ that accounts for the different ejection velocities experienced by the ejecta lofted from different areas within the crater.

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Figure 2A: Target body used in FLAG simulations used for comparison in case studies A and B from [8]. 2B: Target body from simulation compared to case study C [8].

Case Studies: We have compared this method for calculating momentum enhancement to three case studies of asteroid deflection scenarios. The projectile characteristics and resulting $\beta$ and $\Delta v$ from both the original analytic method as well as the numerical values are given in Table 1. The updated values are also given. Cases A and B shared target characteristics and are represented by the object shown in figure 2A. The target for Case C is shown in figure 2B and is a scaled down comet 67P, given a maximum diameter of 200 m. The projectile masses and velocities are given at the top of table 1. For the purposes of finding the peak particle speed in the target following impact, the object from cases A and B is assumed to have impedance matching constants consistent with basalt, while case B was assumed to have constants similar to water ice [7].

Each case was modeled numerically using the Lagrangian hydrocode FLAG [8]. Ejecta that contributed to $\beta$ values in the numerical models met the threshold of having a velocity parallel to projectile’s initial momentum that is greater than escape velocity of object. The ranges for the reported numerical $\beta$ and $\Delta v$ values arise from simulations of the projectile striking the target body on axis and at a normal surface to that initial axis. The analytic approximations are based on pi-scaling relations, which are 1D and do not account for that difference. The final two rows of table 1 show the resulting $\beta$ for each case study assuming an upper bound and lower bound for yield strength in the objects. Cases 1 and 2 are based on asteroid S/2003(65803)1, or Didymos B, and bounded by $25.9 \leq Y \leq 247.4$ MPa. Case C, which is based on the shape model of comet 67P/Churyumov-Gerasimenko scaled to 200 m and composed of dry, porous silicate, is bounded by $40 \ Pa \leq Y \leq 3 \ MPa$. Error in the analytic solution arises from computational rounding and is excluded.

Results: The updated momentum enhancement values are in better agreement with numerical simulations than the prior method but do not account for all contributions of momentum enhancement. The strength parameter study predicts that impacts into low strength objects yield higher $\beta$ than high strength objects.

Discussion: The updated analytic solution for momentum enhancement parameters shows promise, but may not include all of the contributions to momentum enhancement in deflection scenarios. It does, however, provide a more careful treatment of crater ejecta and its contribution to momentum enhancement and resulting velocity change of the target object. It also provides a tool for sensitivity analysis of deflection scenarios, which may inform intensive numerical studies.