PHOTOMETRIC FUNCTIONS AND THE IMPROVED PHOTOCLINOMETRY METHOD: MATURE LUNAR MARE SURFACES. N. V. Bondarenko, I. A. Dulova, and Yu. V. Kornienko, Institute of Radiophysics and Electronics, NAS of Ukraine, 12 Ak. Proskury, Kharkiv, 61085, Ukraine (nbondar@ucsc.edu).

Introduction: LROC NAC images provide an opportunity for studies of the lunar surface properties at spatial scales down to the image resolution. However, interpretation of images themselves is often affected by the surface topography.

In the present work we show application of the improved photoclinometry method for the lunar surface relief retrieval from LROC NAC images. The method is the most mathematically rigorous and allows calculation of the most probable relief based on available images regardless of their number. We study the differences in relief retrieval caused by the use of different a priori known surface photometric functions.

The improved photoclinometry method: Initially, the “photoclinometric” approach involving known dependence of the surface facet brightness on its orientation has been proposed by Van Diggelen [1]. However, that method [1] solves a mathematically incorrectly posed problem (see [2]), which leads to difficulties with its practical application (for example, [3]). The method of improved photoclinometry is based on a statistical approach; it produces the most probable surface height variations consistent with the source images. The accuracy of calculated heights depends on the noise level of source images. Spatial resolution of the topography obtained is limited by the resolution of source images.

First, observed images’ brightness \( I(x,y) \) is recalculated into the slopes field \( t(x,y) \) using a priori known brightness-tilts relation. The gradient of true surface heights \( H(x,y) \) differs from the calculated slopes \( t(x,y) \) due to a random registration noise \( \delta(x,y) \):

\[
\nabla H(x,y) = t(x,y) - \delta(x,y).
\]

The presence of the noise is the reason for the problem formulated in [1] to be mathematically incorrectly posed. Because of \( \delta(x,y) \), equation (1) cannot be solved for \( H \). Instead, we use the Bayesian statistical approach [4] to find the most probable \( H(x,y) \) given measured \( t(x,y) \). Under assumption that the true relief and the image noise are realizations of stationary Gaussian processes, the solution of the problem leads to the Poisson equation with von Neumann boundary condition [2]:

\[
\Delta H(x,y) = \nabla t(x,y).
\]

We applied a finite difference method to solve the Poisson equation (2).

At least two images with different solar azimuths are needed to calculate the heights distribution. Solar azimuths normal to each other are the most preferable observational conditions. The use of three and more images enables retrieval of surface photometric properties, for example, albedo variation, in addition to relief.

Photometric functions: We consider that images’ brightness \( I \) can be presented as

\[
I = A \cdot F(i, e, g)
\]

where \( F(\cdot) \) is a photometric function and \( A \) is surface albedo; \( i, e, \) and \( g \) are incidence, emission, and phase angles, respectively; they completely define the illumination and observation geometry.

To calculate surface heights gradient \( \mathbf{t} \) given measured \( I \), we consider four possible \( F(\cdot) \): (a) Lambert law, \( F = \cos(i) \cdot (LL) \); (b) Lommel-Seeliger law, \( F = 2 \cdot \cos(i) \cdot (\cos(e) + \cos(i)) \cdot (LS) \); (c) McEwen’s model [5] proposed for Clementine photometric correction, \( F = L(g) \cdot LS + [1-L(g)] \cdot LL \), where \( L(g) \) is a function that describes limb darkening [6]; (d) normalized empirical photometric function derived based on LROC NAC observations of mature mare surfaces [7].

Source data: We applied the improved photoclinometry method to a 5.6 × 3.7 km surface area in Mare Imbrium. We processed three images, which were portions of LROC [8] images M183661434LE (Fig. 1a), M186020337RE, and M188379229LE [9]. These images were taken when solar azimuths (from the North clockwise) \( A_s \) were equal to 54.81º, 17.13º, and -32.99º, respectively. One images pair gives the difference between \( A_s \) of 87.8º. Images were transformed to the same resolution of 5.25 m/pixel and co-registered to each other. Initial surface albedo was calculated at every \((x, y)\) location as an average of three estimates obtained from the three images according to Eq. (3) using assumed photometric function and proper observational conditions \( i, e, g \).

Results: Topography of the area under study calculated from three source images and empirical photometric function \( d \) as a priori known one is shown in Fig. 1b. Statistics for this relief is listed in Table 1 along with data for the topography calculated using other photometric function under study.

According to Table 1, the relief obtained with the empirical photometric function \( d \) is very close to one based on Lommel-Seeliger law (b) since the difference between them \( \Delta H \) is the lowest in comparison to (a) and (c) cases.

In general, the relief retrievals with all photometric functions under study are qualitatively similar to each other. It is seen in Fig. 2, where elevations plotted along pink line (Fig. 1b) from the left (west) to the...
right (east). Empirical function (d)-based topography (red line, Fig. 2) is very close to one based on Lommel – Seeliger law (b) (blue line, Fig. 2). Lambert law (a) (black line, Fig. 2) looks the most unsuitable for the purpose of the relief retrieval from images in mature mare areas.

Fig. 1. Lunar surface area under study (~5.6 × ~3.7 km, center coordinates: 32.48ºN, 340.12ºE): (a) portion of M183661434LE image, illumination direction is marked with yellow arrow; (b) calculated relief, darker shades correspond to lower heights.

Taking into account retrieval relief we recalculated the surface albedo distribution with all photometric functions under study. Some properties of these albedo distributions are shown in Table 1.

<table>
<thead>
<tr>
<th>Function (F)</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
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<tbody>
<tr>
<td>$H$, m</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>min</td>
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<td></td>
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</tr>
<tr>
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<tr>
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<td>4.76</td>
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<tr>
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</tr>
<tr>
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<tr>
<td>max</td>
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<td>0.052</td>
<td>0.054</td>
<td>0.053*</td>
</tr>
</tbody>
</table>

* - assuming the mean albedo of mature mare to be 0.08; “min”, “max”, “avr” – minimum, maximum, and average values; $\sigma$ - the root-mean-square deviation.

Fig. 2. Retrieval heights variations $H$ vs. profile (pink line, Fig. 1b) length $L$ based on photometric functions (a) – (d).

Conclusions: The application of the improved photoclinometry method for the relief retrieval from images shows adequate results for the lunar mare surface area. Lommel – Seeliger law (isotropic single scattering model) appears to be the most preferred photometric function for the purposes of the relief retrieval in mature mare areas.