Seismic Waves in the Asteroid Environment Paul Sánchez[1] and Daniel J. Scheeres[2], [1]Colorado Center for Astrodynamics Research, University of Colorado Boulder, 3775 Discovery Dr, Boulder, CO 80303, [2]Ann and H.J. Smead Aerospace Engineering Sciences, University of Colorado Boulder, 3775 Discovery Dr, Boulder, CO 80303 (diego.sanchez-lana@colorado.edu).

By means of numerical simulations we investigate impact generated seismic wave transmission in granular media under extremely low pressure. This mimics the conditions in the interior of asteroids. We find a dependency not only on the overburden pressure on the medium, but also on the energy of the impact.

Introduction: Experimentally, it is known that the speed of seismic waves in a granular medium is related to the overburden pressure (p) applied to it [1, 2]. The theory that has been developed is called Effective Medium Theory (EMT) and is based in considering the media as elastic, thus removing the difficulties presented by particle size and shape. Based on this assumption, it is possible to calculate, from first principles, the bulk and sheer moduli and from them, the speed of sound. This theory, based on the Hertzian laws for contacts, predicts a sound velocity that has a $P^{\frac{1}{6}}$ dependence on pressure (P), whereas the experiments have found this is true only for high enough pressures and that at low pressures the dependency changes to $P^{\frac{1}{4}}$. However, as far as we are aware, experiments have been limited to the kPa pressure regime as they have been carried out under Earth's gravity. Additionally, seismic wave propagation has been investigated towards understanding process that take place on terrestrial soils and so MPa and GPa of pressure are necessary. However, the interior of small Solar System bodies are subjected to very different conditions.

These small bodies (asteroids, comets and small moons) produce gravitational fields in the *milli-* and *micro-g* regimes and, as a consequence, their interior pressure vary from zero (on their surface) to just a few Pascals or tens of Pascals in their innermost regions. This being so, it would be easy to see that the resulting wave speed should be minuscule if what is known about sound wave transmission on Earth can be carried over to small planetary bodies. It would not be surprising to find a maximum speed in the range of only a few tens of m/s or even less than that.

If this is so, an impact, such as the one produced by the JAXA Hayabusa2 mission on asteroid Ryugu (300 m/s) should be seen as at hyper-velocity though without any of the comminution and chemical processes that this type of impacts are usually associated with. The same reasoning should be valid for the TAGSAM manoeuvre (10-20 cm/s) of the NASA OSIRIS-REx mission on asteroid Bennu and the impact test (6-7 km/s) by the future NASA DART mission.

In order to investigate seismic wave transmission in the asteroid environment, we carry out granular dynamics simulations that mimic these low pressure conditions. We use an in-house developed soft-sphere discrete element method (SSDEM) code and relate our findings to the missions described above.

Soft-Sphere DEM: The simulation program that is used for this research applies a Soft-Sphere Discrete Element Method (SSDEM) [3, 4], implemented as a computational code to simulate a granular aggregate [5, 6, 7, 8]. The particles, modelled as spheres that follow a predetermined size distribution, interact through a soft-repulsive potential when in contact. This method considers that two particles are in contact when they overlap. When this happens, normal and tangential contact forces are calculated [9]. The former is modelled by a hertzian spring-dashpot system and is always repulsive, keeping the particles apart; the latter is modelled with a linear spring that satisfies the local Coulomb yield criterion. The normal elastic force is modelled as

$$\vec{\mathbf{f}}_e = k_n \xi^{3/2} \hat{\mathbf{n}}, \tag{1}$$

the damping force as:

$$\vec{\mathbf{f}}_d = -\gamma_n \dot{\xi} \hat{\mathbf{n}}, \tag{2}$$

and the cohesive force between the particles is calculated as

$$\vec{\mathbf{f}}_c = -2\pi \frac{r_1^2 r_2^2}{r_1^2 + r_2^2} \sigma_{yy} \hat{\mathbf{n}} \tag{3}$$

where r_1 and r_2 are the radii of the two particles in contact, σ_{yy} is the tensile strength of this contact, which is given by a cohesive matrix formed by the (non simulated) interstitial regolith [10], and $\hat{\mathbf{r}}_{12}$ is the branch vector between the centres of these two particles. Then, the total normal force is calculated as $\vec{\mathbf{f}}_n = \vec{\mathbf{f}}_e + \vec{\mathbf{f}}_d$. In these equations, k_n is the elastic constant, ξ is the overlap of the particles, γ_n is the damping constant (related to the dashpot), $\dot{\xi}$ is the rate of deformation and $\hat{\mathbf{n}}$ is the vector joining the centres of the colliding particles. This dashpot models the energy dissipation that occurs during a real collision.

The tangential component of the contact force models surface friction statically and dynamically. This is calculated by placing a linear spring attached to both particles at the contact point at the beginning of the collision [9, 11] and by producing a restoring frictional force $\vec{\mathbf{f}}_t$. The magnitude of the elongation of this tangential spring is truncated in order to satisfy the local Coulomb yield criterion $|\vec{\mathbf{f}}_t| \leq \mu |\vec{\mathbf{f}}_n|$.

Procedure: For this particular set of simulations, we have chosen to use material parameters close to those of asphalt so that the results are relevant for asteroids. We have chosen to simulate perfectly spherical grains so that the results can also be related to sound transmission theory (as explained in the previous sections). We use 3000 grains with diameters between 2-3 cm that follow a uniform, random distribution. Their density is 3200 kg m^{-3}, Young modulus is 7.8×10^{10} N m^{-2}, Poisson ratio is 0.25 [12]. Based on these parameters, the values of k_n and γ_n can be calculated and adjusted in order to manipulate the coefficient of restitution of the particles. In our simulations, we have chosen to use two coefficients of restitution: 0.1 while the particles are settling and 0.5 for the actual simulations. This reduces the settling time.

The particles are contained in box with a solid bottom, horizontal periodic boundary conditions and a moving top that allows us to impose a very well determined pressure to the system. Initially, the particles are placed inside this box so that their centres form a hexagonal close packed lattice (HCP), then they are given random speeds in all three axes of motion ([-0.15, 0.15]m/s) and let to settle under Earth's gravity. Once that is done, the moving top is place on the surface of the grains and the system is allowed to settle again. Finally, the weight of the entire system is calculated, gravity is set to zero and a force equal to that weight is applied to the moving top so that the system is minimally disturbed and it is allowed to settle again. This force is diminished by one order of magnitude and the system is allow the settle for as many times as needed to reach the desired over burden pressure. During this process, the particles are also subjected to a Stokes' like drag so that the energy released when the pressure is reduced is quickly removed. The height of the settled system is approximately 80 cm (see Fig. 1).

In order to initiate the seismic wave, all the particles below a height of 1.5 times the average radius are given a vertical velocity (kick) of 0.5 m/s after 0.1 s from the moment the settling procedure is finished. The entire system is then divided into horizontal slices 5 cm thick so that more than a monolayer is observed at a time. The kinetic energy of each slice is calculated in order to observe the wave transmission. Data is collected every 1×10^{-4} s after the wave is started and this is done for 0.03 s which gives enough time for the wave to go through the system completely. The sound speed is measured as the ratio between the time that takes the peak of the wave to reach the 15th slice and the height of this slice. The overburden pressure was varied between 0.1 and 50000 Pa, whereas σ_{yy} varied between 0 and 1000 Pa.

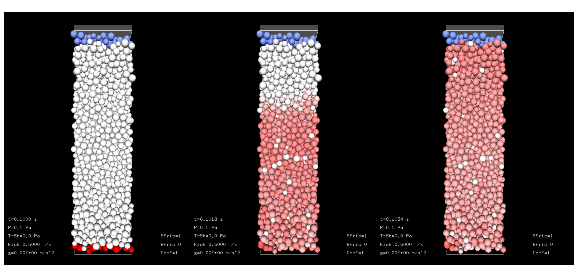

Figure 1: Simulation setup, the redder the particle, the greater its vertical speed component (P=0.1 Pa).

Results: Though the simulations correctly predict the wave speeds at high pressures (>10 kPa), these speeds plateaued at lower pressures (≈140 m/s), which was not expected. The addition of cohesive forces between the particles was not seen to produce any appreciable effect in the investigated range. This prompted us to change the initial vertical velocity of particles that generated the pressure wave to 0.01, 0.1 and 5 m/s. This produced wave speeds that also changed with pressure but plateaued at a lower values (≈42, 91 and 234 m/s respectively). This has prompted us to believe that the change in the local pressure produced by the passing of the wave is responsible for its velocity. At high pressure, the additional pressure provided by the passing of the wave would be inconsequential, but at lower pressures, such as the one existent in small bodies, it is determining. Additional details will be provided at the conference.

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