TIDAL PUMPING OF WATER THROUGH THE POROUS CORE OF ENCELADUS

Y. Liao[1], F. Nimmo[2], J.A. Neufeld[3] [1] Woods Hole Oceanographic Institute (yliao@whoi.edu), [2]University of California Santa Cruz (fnimmo@ucsc.edu) [3] Cambridge University (jn271@cam.ac.uk)

Introduction: Gravity and topography measurements suggest that the rocky core of Enceladus has a low density [1], probably as a result of high porosity [2]. Such a core is potentially deformable enough to generate significant tidal deformation and heat in the porous solid [2,3]. Here we will investigate an additional physical effect: tidal pumping of fluid (water) through the porous core. Viscous dissipation associated with fluid moving back-and-forth through narrow pores generates heat, but little attention has been paid to this effect in the literature [4]. Both the source of the ∼10 GW of heat measured at the tiger stripes of Enceladus [5] and the long-period variability of the Enceladus plume [6,7] represent major unsolved puzzles. The preliminary results presented here suggest that tidal pumping of fluid may be important in terms of Enceladus's overall energy budget, and its tidal response at long periods.

Theory: We adopt a poroviscoelastic framework [8] in which the porous solid is assumed to be Maxwell viscoelastic [9]. The constitutive relationship is:

$$\left(\frac{\partial\underline{\sigma}}{\partial t}+\frac{\mu}{\eta_m}\underline{\sigma}\right)-\frac{1}{3}\frac{\mu}{\eta_m}Tr(\underline{\sigma})\underline{I}=2\mu\frac{\partial\underline{\varepsilon}}{\partial t}$$
$$+(K_m-\frac{2}{3}\mu)\frac{\partial Tr(\underline{\varepsilon})}{\partial t}\underline{I}-\alpha\frac{\partial P_f}{\partial t}\underline{I} \qquad (1)$$

where $\underline{\sigma}$ and $\underline{\varepsilon}$ are the total stress and strain tensors, μ and η_m are the rigidity and viscosity of the rocky matrix, P_f is the pore fluid pressure and K_m is the "drained" bulk modulus, which is related to the bulk modulus of the pore-free solid K_s by $K_m=(1-\alpha)K_s$, where α is the so-called Biot coefficient which ranges from 0-1 [8,10]. The Biot coefficient describes the degree to which the porous solid is weaker than its pore-free equivalent. The final term on the RHS of this equation couples the stress in the solid to the behaviour of the fluid.

The fluid behaviour is described by two further equations. The first concerns the relationship between pore fluid and volume:

$$\zeta=\alpha\left(\frac{\alpha}{K_u-K_m}P_f+\underline{\nabla}\cdot\underline{u}\right) \qquad (2)$$

Here ζ is the volume of fluid in the pores per unit volume of undeformed material, K_u is the "undrained" bulk modulus and u is the solid deformation. This equation shows that an increase in the volume of fluid contained within the pores arises from either an increase in pore fluid pressure or an increase in overall volume.

Finally, the motion of fluid in response to pressure changes is given by Darcy's law:

$$v_d=-\frac{\kappa}{\eta_f}\underline{\nabla}P_f \qquad (3)$$

where v_d is the fluid Darcy velocity, η_f the fluid viscosity and κ the permeability.

Analytical Solution: This set of equations may then be solved, together with appropriate boundary conditions and conservation of mass, to determine the response of the fluid in the core to tidal forcing. Here we will assume that the forcing is periodic. For this preliminary analysis we assume a maximum strain amplitude ε_0, allowing simple comparison with the equivalent results for a solid body, and a (2,0) spatial pattern.

The depth-dependence of the flow is given by a characteristic wavenumber, k, which depends on the forcing period and the material properties of the core and fluid:

$$k^2=-\frac{\omega i}{c_0}\frac{K_u(K_m+\frac{4}{4}\mu)+i\omega\tau(K_m+\frac{4}{3}\mu)}{K_m(K_u+\frac{4}{3}\mu)+i\omega\tau(K_u+\frac{4}{3}\mu)} \qquad (4)$$

where ω is the forcing frequency, $\tau=\eta_m/\mu$ is the solid Maxwell viscoelastic relaxation time [9], and c_0 is a diffusivity given by

$$c_0=\frac{\kappa}{\eta_f}\frac{(K_m+\frac{4}{3}\mu_0)(K_u-K_m)}{(K_u+\frac{4}{3}\mu_0)\alpha^2} \qquad (5)$$

This expression shows that fluid pressure diffuses more rapidly with higher permeability or lower fluid viscosity, as expected; the rigidity of the solid matrix also plays a role, because a change in total volume will also cause fluid motion. In the limit of an incompressible matrix we recover a diffusivity of $\kappa K_f/\eta_f\phi_0$ which (with K_f the fluid modulus and ϕ_0 the reference porosity) is a standard permeable flow result [4].

The amplitudes of the parameters of interest (stress, fluid velocity etc.) may then be derived analytically. For instance, the amplitudes of the radial strain, fluid pressure and radial Darcy velocity are, respectively:

$$\hat{\varepsilon}_{rr}=(3\cos 2\theta+1)\left(-\frac{\alpha C_1}{k^2(K_m+\frac{4}{3}\mu^*)}\frac{d^2j_2(kr)}{dr^2}+2C_2\right)$$

$$\hat{P}=C_1(3\cos 2\theta+1)j_2(kr)$$

$$\hat{v}_{d,r}=-\frac{\kappa}{\eta_f}C_1(3\cos 2\theta+1)\frac{dj_2(kr)}{dr}$$

where C_1 and C_2 are constants determined by the boundary conditions, j_2 is a degree-two spherical Bessel function and θ is colatitude. With our analytical solutions for $\underline{\sigma},\underline{\varepsilon},\cdots$ we can then determine the time-averaged heat production rate due to fluid flow:

$$\dot{E}_{perm.}=\frac{1}{T}\int_0^T\frac{\eta_f}{\kappa}(\underline{v}_d\cdot\underline{v}_d)\,dt=\frac{1}{2}\frac{\eta_f}{\kappa}(|\hat{v}_{d,r}|^2+|\hat{v}_{d,\theta}|^2)$$

with T the forcing period.

Preliminary Results: Below we discuss our preliminary results in terms of heat production and dependence on forcing period

Heat Production. Figure 1 below shows how permeable flow affects the overall core heating rate. The dashed line shows the variation in heat production with solid viscosity η_m. As expected, the heating peaks when the Maxwell time is comparable to the forcing period [9]. The red and blue lines show the effect of including heating arising from permeable flow, with different choices of Biot parameter α and permeability κ. At high solid viscosities there is little solid-body heating, because the deformation is almost elastic. This deformation, however, still drives fluid flow, which produces heat. The heating is higher if the permeability is larger (larger Darcy velocity) or the weakening effect of the porosity is more pronounced (larger α). Notably the total contribution to heat with a permeability of 10^{-8} m^2 can exceed 10 GW, suggesting that permeable flow could be a major heat source for suitable parameter choices.

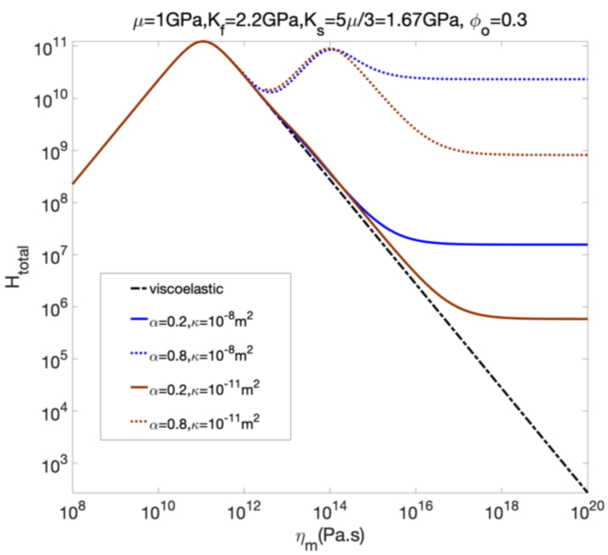

Figure 1 Total core heating rate (in W) as a function of solid core viscosity η_m for differing combinations of Biot parameter (α) and permeability (κ). Here a solid rigidity of 1 GPa is assumed.

Figure 2 shows the spatial distribution of heat production for the 10^{-8} m^2 permeability case. The heat production is strongly concentrated in the polar regions and in the near-surface of the core.

Frequency-dependence The characteristic wavenumber k depends on the forcing frequency (eq. 4). A longer period will result in a larger volume in which significant flow is occurring, but the lower frequency will reduce the volumetric tidal heating. As a result, fluid advection of pre-existing heat may be more important than the direct heating effect of fluid flow at long periods. The reason for considering long-period effects is that the plumes of Enceladus exhibit variability on 4- and 11-year periods [7], strongly suggesting that long-period librations (which have amplitudes comparable to the diurnal tidal forcing [11]) are responsible.

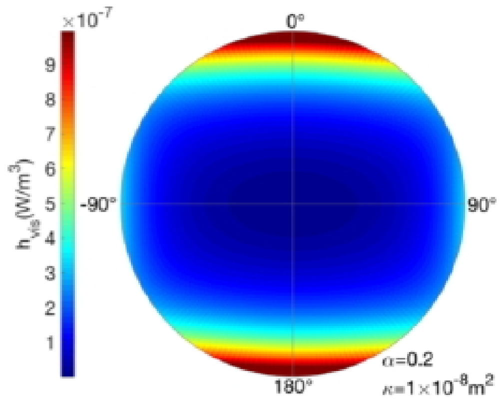

Figure 2 Spatial pattern of permeable heat production in the core. Here an axisymmetric (2,0) forcing pattern is assumed, and 0° represents the north pole.

Future Work: Our preliminary results use simplified boundary conditions and a single degree-2 pattern. Future work will remove both these simplifications, will consider the range of likely parameter values (e.g. permeability) and will consider both advection of pre-existing heat and the tendency for permeable-flow heating to produce hydrothermal circulation [3,12] and water-rock interactions. The frequency-dependence and spatial pattern of the resulting heat production may then be compared with observational constraints [5,7,13] to test whether the physical effects proposed here are relevant to Enceladus.

Conclusions: We have derived analytical solutions for tidally-forced fluid flow and heat production in the permeable core of Enceladus. Our preliminary results (Figure 1) suggest that the heat production may be an important contributor to the overall energy budget, but more work is required to fully test this hypothesis.

References: [1] Iess, L. et al., *Science* 344, 2014. [2] Roberts, J.H. *Icarus* 258, 2015. [3] Choblet, G. et al., *Nature Astron.* 1, 2017. [4] Vance, S. et al. *Astrobiology* 7, 2007. [5] Howett C.J.A. et al. *JGR-P* 2011. [6] Ingersoll , A.P. et al. *Icarus* 2019. [7] Porco et al., this mtg [8] Wang H.F. *Theory of linear poroelasticity*, 2000. [9] Ross & Schubert *Icarus* 78, 1989. [10] Segall P. *Earthquake and volcano deformation* 2010. [11] Rambaux N. et al., *GRL* 2010. [12] Travis & Schubert, *Icarus* 250, 2015. [13] Hemingway, D et al., in *Enceladus and the icy moons of Saturn*, 2018.