FORWARD MODELING PLANETARY INTERIOR STRUCTURE FROM GRAVITY USING GENETIC ALGORITHMS. S.T. O'Hara[1,2] and A.J. Dombard[1], [1]Univ. of Illinois at Chicago, 845 W Taylor St., Chicago IL 60607. [2]Now at Lunar and Planetary Institute, USRA, 3600 Bay Area Blvd., Houston TX 77058, USA. <so-hara@lpi.usra.edu>.

Introduction: Traditional modeling of a planet's interior structure combines constraints on bulk mass, density, triaxial shape, and rotation with observations of gravitational potential field, which are most often expressed in terms of degree-2 spherical harmonic coefficients [1]. The term that is most often dominant in the spherical harmonic expansion is known as the J_2 term, which is a zonal (latitudinal) term that describes a body's gravitational potential due to its oblate shape. For a tidally deformed body, determining the internal distribution of mass additionally requires the C_{22} coefficient. Determining uniquely these coefficients requires a model of the variation in a body's gravitational potential with both latitude and longitude, which in turn can only be constructed from data collected when the observing spacecraft's trajectory crosses the equator.

However, in situations where equatorial crossing is not possible due to mission design or architecture, it is still possible to separate J_2 and C_{22} (and thus analyze the interior structure) by assuming that the internal shape of the body is in hydrostatic equilibrium. This assumption connects J_2 and C_{22} by:

$$\frac{J_2}{C_{22}} = \frac{10}{3} \text{ [2].} \qquad (1)$$

This imposed restriction allows one to inverse model a density structure, but only by constraining the parameters of the model to the hydrostatic assumption. Any spherical harmonic solution for a particular potential field is not unique, so if a non-hydrostatic component is present or if the density structure is unusual, then the inferred interior structure may be inaccurate.

An alternative approach would be to forward model the density structure. This procedure would generate a series of notional density structures, determine the gravitational potential (and spherical harmonic coefficients) that each would produce, and find the model that best matches the observations (line-of-sight accelerations). This forward approach would allow unique determination of the J_2 and C_{22} coefficients while bypassing the hydrostatic assumption. However, forward modeling presents a significant challenge due to the non-unique nature of gravity analysis. Many different interior structures may be reasonable fits for the data. There is thus demand for a tool that can efficiently sort through a large parameter space.

Methodology: We describe a methodology for efficiently forward modeling the internal density distribution of a planetary body utilizing genetic algorithms. Genetic algorithms (henceforth GA) are a method of optimization wherein solutions are ranked by their fitness compared to a defined objective. Individual sets of parameters representing a solution are called a genome. During each generation, the GA generates a population of genomes, which are evaluated compared to the objective. Parameters (genes) within a genome can be transmitted to other members of the population or randomly mutated, allowing the algorithm to avoid converging on spurious local error minima [3].

In this formulation, the objective function can be the directly observed line-of-sight gravity acceleration residuals of the spacecraft, while the genome is a triaxial shape and density structure model of the body. The genome is expanded to its gravity potential field, which can be directly compared with the objective data. The objective function and genome selected is effectively arbitrary, and can also be used to calculate and compare spherical harmonic coefficients. This method implementation is currently under development as an open source C++ code tentatively named GFGA (Gravity From Genetic Algorithms).

Model formulation: The first step in the forward model is to define the parameters of the planetary body that will form an individual genome. We adopt the method of Kattoum and Dombard [4] by treating the body as a series of overlapping triaxial ellipsoids, each with radii that extend to the center of the body. The effective density of each successively inward body is the density contrast between the actual physical layers. While unphysical in real terms (multiple bodies occupy the same space), this calculation of the potential can be shown to produce the same result as the classical formulation. For a hypothetical three-layer body, the potential U at a vector position $\boldsymbol{x}$ outside the body is given by:

$$U(x) = -\int_0^{V_1} \frac{d^3x' G\rho_1}{|x-x'|} - \int_0^{V_2} \frac{d^3x' G\rho_2}{|x-x'|} - \int_0^{V_3} \frac{d^3x' G\rho_3}{|x-x'|}, \quad (2)$$

where V_1, V_2, and V_3 are the volumes of a series of 3 sequentially enclosed bodies, and ρ_1, ρ_2, and ρ_3 represent the density *contrast* between each body and the preceding body that encloses it. Note that in the case of V_1, this is the contrast between it and the zero-density space outside of the body (i.e., the absolute density of the outer most layer). Eq. 2 is easily extendable to an arbitrary number of layers. An alternative approach would be to consider the planetary body as a suite of voxels, permitting the inclusion of lateral (and not just radial) density variations.

Each ellipsoid is meshed according to a longitude/latitude grid of variable resolution (between 1 and 10 degrees depending on the desired balance of accuracy and solution time). This mesh is then tessellated into a series of triangular plates. Following Cheng et al. [5], gravity elements are calculated by treating these plates as prisms that extend to the center of the body and integrating along a vector running through the plate's centroid to a point on the reference sphere outside the body. If a center of figure offset is present in the body, it is applied as an offset to these vectors relative to the reference sphere. The result is a field representing the gravitational potential at a series of points on the reference sphere. Each point accounts for the gravitational pull on it from every plate-prism within the body.

Genetic algorithm: The software for this work utilizes the GAlib genetic algorithm package [6]. GAlib is a freely available source code that provides the C++ framework for the construction and evaluation of genetic algorithm codes. GAlib supports overlapping (steady-state) and non-overlapping (simple) populations, multiple replacement or selection methods, and fully customizable initialization, mutation, and crossover functions, as well as evaluation functions with either population or individual-based evaluation.

The procedure of the GA is as follows: a population of genomes, consisting of the triaxial shape and density contrast of a set of overlapping ellipsoids, is created. The code then calculates the derived parameters (gravity potential field, spherical harmonics, and bulk density) of each genome. These solutions are evaluated and ranked by an difference function compared to the objective (line-of-sight gravity residuals or spherical harmonics). Genomes with values close to the objective are ranked higher, and are thus more likely to be passed on to the next generation; physically unreasonable solutions are discarded. The genomes are then stochastically cross-bred and mutated to form the population for the next generation. The GA runs until a defined convergence value is reached, or for a defined number of generations.

As noted, the number of layers within a body can be effectively arbitrary, allowing the user to define the resolution of the density gradient. However, caution should be used, as each additional layer requires an additional meshing operations for each individual in the population, increasing computation requirements by the product of the mesh resolution and the population size (variable, but at least ~10^5). For the same reason, allowable ranges of density and ellipsoid dimensions should be carefully considered to prevent unphysical or unlikely model arrangements from consuming excessive computation time. Advances in computer power in the future will reduce this burden.

Applications: The hydrostatic assumption (1) is required to inverse model the structure Jupiter's moon Europa, due to the high gravity and challenging radiation environment that has thus far precluded an equatorial crossing mission architecture [7]. Europa has both an ocean underneath its ice shell and a source of tidal heating, thus making it a prime target for astrobiological exploration [8,9]. Current inverse modeling methods are not able to determine uniquely the thickness of the ice shell relative to the liquid water ocean [2]. Bypassing the hydrostatic assumption via forward modeling provides a potential (pun intended) way to improve discrimination of density variations within Europa's interior that can be directly compared to observations made by NASA's Galileo spacecraft and the planned Europa Clipper mission.

Benchmarking efforts are currently underway to improve the functionality of GFGA in preparation for analysis of Europa observational data. The code is currently benchmarked for single-layer, constant density determination of gravity potential and spherical harmonics. It can also reach solutions within the parameter space defined by Anderson et al. [2] when given the bulk shape and density of Europa. Benchmark functions for the gravity potential of multi-layered bodies, as well as line of sight gravity potential input, is currently under development, after which the code is planned to be released as an open source tool for the community.

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