COMPARATIVE PLANETOLOGY: SCALING LAWS OF TOPOGRAPHY ON EARTH, MARS, MOON, MERCURY (AND EXOPLANETS ?) F. Schmidt[1], F. Landais[1], S. Lovejoy[2]
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Summary: We investigate the scaling properties of the topography of the Earth, Mars, the Moon and Mercury at global scale down to the highest available scale. Planetary topographic fields are well known to exhibit (mono)fractal behavior. Still, a single fractal dimension is not enough to explain the huge variability and intermittency of these fields. Previous studies have shown that fractal dimensions might be different from a region to another, excluding a general description at the planetary scale. In this project, we are analyzing the topographic datasets with a multifractal formalism to study the scaling intermittency. In the multifractal paradigm, the local variation of the fractal dimension is interpreted as a statistical property of multifractal fields. The results suggest a multifractal behavior from planetary scale down to 10 km. From 10 km to 100 m, the topography seems to be simple monofractal. This transition indicates a significant change in the processes governing the planetary surfaces. Using a comparative planetology approach, we will bring new elements to discuss the origin and formation of the telluric bodies.

Introduction: The acquisition of altimetric data from Mars Orbiter Laser altimeter (MOLA) has motivated numerous analysis of the Martian topography, in particular the surface roughness. A possible approach is to assume that topography can be mathematically described as a statistical field with quantitative parameters able to characterize the geological units. Many statistical indicators have been proposed and widely explored in order to study the surface of Mars: RMS height, RMS slope, median slope [1], autocorrelation length [2]. Useful information has been obtained by the use of those indicators but they have the disadvantage of been defined at a given scale. By construction, they do not directly take into account the well-established scale symmetry that generally occurs in the case of natural surfaces. Indeed, statistical parameters like the mean or the standard deviation exhibit dependence toward scales. Hence the nature of this dependence needs to be accurately described, otherwise the description of the surface remain incomplete. This subject has been widely studied in the past, parallel to the development of the notion of fractals [3]. More interestingly, the fractal theory provides a mathematical formalism to describe the scale dependence of statistical parameters toward scales. It turns out that simple power-law relations efficiently approach the variability of planetary surfaces. The associated power-law exponent provides a quantitative parameter that is a good scale-independent candidate to characterize the geometric properties of a natural surface. A common example is given by the power spectrum of topographic field providing roughness information in the frequency space as done locally for the Moon [4].

On Mars, different authors have explored the scaling properties of topography by the use of scale invariant parameters. The observed local variation [5] apparently rejects the idea of a global description of any topographic field at the planetary scale. However, modern developments in the fractal theory might be able to give full account to the observed variability and intermittency. As proposed by [6], it is possible to extent the fractal interpretation of topography to a multifractal statistical object requiring an infinite number of fractal dimensions (one for each statistical moment).

Dataset: We used the MOLA instrument database to study Mars [7], LOLA for the Moon [8], MLA for Mercury [9] and ETOPO1 for the Earth [10]. The highest resolution varies from 60 m from LOLA to 1853 m for the Earth and the number of roughness measurement (fluctuations) varies from 2.10^5 for MLA to 1.10^{10} for LOLA.

Method: To define a fluctuation (a local roughness measurement), the simple altitude difference can be used. It corresponds to the “poor man” wavelet and can be advantageously replaced by the Haar wavelet that is more accurate and is useful over a wider range of exponents ($-1 < H < 1$ rather than $0 < H < 1$ for differences) [11]. Then the mean Haar fluctuation over the entire planet, depending on scale is represented to exhibit scaling laws. In order to decipher monofractal versus multifractal, we also compute different moments (for instance : average of the square, for the moment order 2). This way, we can capture the full statistical distribution of roughness.

Results: Figure 1 shows the main result of this analysis and demonstrates that scaling laws are appropriate to describe the topographic fields of planetary bodies. Although the linear correlation (scaling) is satisfying for all planetary bodies, two distinct scaling regimes occur with a transition around 10 km. At scales larger than 10 km, all planetary bodies are different. Interestingly, the scaling law is characterized for the Moon by H = 0.2, Mercury by H = 0.3, Mars and Earth by H = 0.5. At scale smaller than 10 km, the topography is still scaling but H = 0.8.

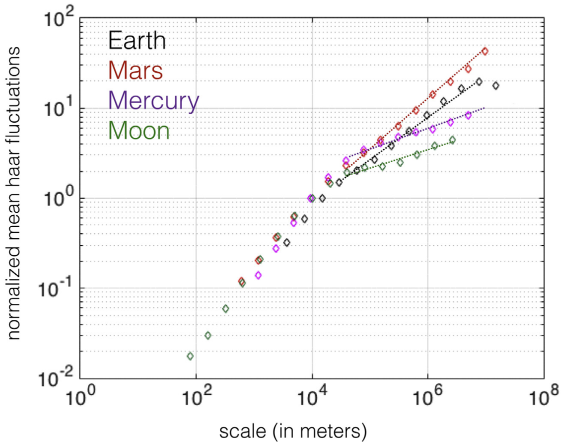

Figure 1: Mean Haar fluctuations normalized in order to be approximatively equal at scale 10 km, as a function of scale. The normalization does not modify the scaling behavior but emphasize the transition that seems to occur around 10 km.

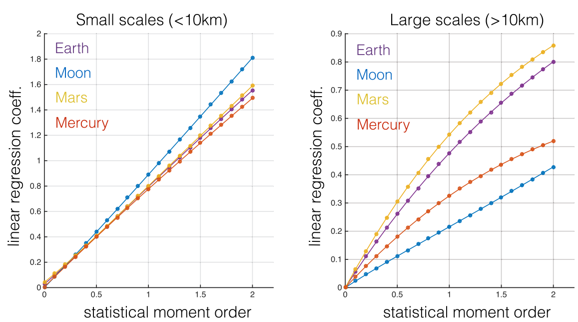

Figure 2: Structure function for different ranges of scales.

In addition, Figure 2 presents the slope of the linear fit for different moments. At low scale (< 10 km), figure 2 (left) indicates a simple linear relationship, demonstrating a monofractal behavior. The curvature on figure 2 (right) indicates that multifractal scaling seems to occur on a large but restricted range of scale (> 10 km). The curvature is described by the C1 parameters. Whereas the case of Mars, Mercury and Earth have similar of values of C1 around 0.1, the case of the Moon seems to be an exception with weak multifractal properties over the whole range of scales (C1 close to 0).

Discussions: We demonstrate that a change of processes governing the planetary topography occurs at 10 km [12]. A multiplicative cascade process is occurring at scale higher than 10 km but a simpler monofractal scaling process is occurring a small scale. The same transition occurs for the Earth, Mercury and the Moon [13]. We propose the interpretation that the elastic thickness of the lithosphere is responsible for this transition by acting against the deformations caused by the different surface processes in two regimes. The value of H may be related to its geological activity.

The smaller the body, the less intense its internal activity due to intense thermal cooling. The value of H may be related to its geological activity. One can speculate that a more intensively convecting mantle yields a higher value of H.

Craterisation is well known to be a fractal process with a single fractal dimension [4]. We propose that the low scales are dominated by craterisation processes, at the origin of the monofractal scaling law, as suggested [1]. Most probably, other effects, such erosion and volcanism, should be dominant at larger scales.

If the multifractal laws are fundamental of planetary topography, we can generate synthetic topographies for the exoplanets with multifractal behavior [14]. An online tool allows us to explore such distant worlds in 3D [15].

References:

[1] O. Aharonson, et al. JGR : Planets, 106(E10):23723–23735, 2001. [2] M. A. Kreslavsky and J. W. Head. JGR: Planets, 105(E11):26695–26711, 2000 [3] B. Mandelbrot, Science, 156(3775):636–638, 1967. [4] M. A. Rosenburg, and O. Aharonson. JGR: Planets, 2015. [5] R. Orosei, et al. JGR: Planets , 108, 2003 [6] S. Lovejoy and D. Schertzer, JGR: Atmospheres, 95(D3):2021–2034, 1990. [7] D. E. Smith et al, JGR: Planets, 106 (E10):23689–23722, 2001. [8] D. E. Smith et al., (2010) Space Science Review, 150, 209-241 [9] J. F. Cavanaugh et al., (2007) Space Science Review, 131, 451-479. [10] C. Amante et al., (2009) NOAA Technical Memorandum NESDIS NGDC-24 [11] S. Lovejoy and D. Schertzer. Nonlinear Processes in Geophysics, 19(5):513– 527, 2012. [12] Landais, F., Schmidt, F. & Lovejoy, S. (2015), Nonlinear Processes in Geophysics, 22, 713-722 [13] F. Landais., F. Schmidt. & S., Lovejoy (2019), Icarus, 319, 14-2 [14] F. Landais., F. Schmidt. & S., Lovejoy (2019), Monthly Notice of the Royal Astronomical Society, 484, 787-793 [15] https://data.ipsl.fr/exotopo/