TOWARD SELF-CONSISTENT-FIELD MODELS OF THE MOON AFTER THE FORMATION IMPACT (II) . D. G. Korycansky, *CODEP, Department of Earth and Planetary Sciences, University of California, Santa Cruz CA 95064* .

Introduction

In recent years, the canonical picture of the most widely-accepted scenario of the Moon's formation (the giant impact theory) has become complicated due to recent high-precision measurement of isotope ratios for oxygen and other elements in lunar samples. The measurements suggest that the bulk isotope ratios for the Moon are essentially identical to those of the Earth (e.g. Zhang *et al.* 2012). This is a problem for the "classic" scenario in which the Moon is formed by the oblique impact of a Mars-mass body with the proto-Earth and subsequent accretion from a post-impact disk surrounding the Earth. The classic picture suggests that the Moon is primarily made of impactor material. Generating near-identical isotope ratios by this sequence of events would require a presumably unlikely near-identical isotopic composition of the impactor and proto-Earth.

Thus, new models have been proposed that may overcome this difficulty. In particular, Ćuk and Stewart (2012) proposed a model in which the proto-Earth is rapidly spinning and struck by a relatively low-mass impactor, forming a hot, rapidly rotating structure in which convection allows effective mixing and homogenization of isotope ratios for the subsequently formed Moon. This type of post-impact structure has been dubbed a "synestia" by Lock et al. (2017, 2018).

The Self-Consistent Field method

The work described here is part of a larger effort to validate formation models via high resolution simulations the Moon-forming impact with a state-of-the-art hydrodynamics code. While such a calculation is the best method available for describing the impact and its immediate aftermath, it is limited to modeling a time period of hours to several days after the impact. Studying the longer-term evolution of the Earth-Moon system will require a different method that sacrifices some of the dynamical details of the system in order to model quasi-steady-state long-term development. We have been working on applying the so-called Self-Consistent Field (SCF) method in order to generate post-impact configurations. The SCF method has a long pedigree in astrophysics stretching back to the 1960s, having been applied to modeling self-gravitating bodies with significant angular momentum (Tassoul 1979). In particular it is applicable to the "in-between" situation where objects neither rotate slowly enough to be considered quasi-spherical, or so fast as to be approximated by thin disks (as in accretion disk theory).

Use of the method requires the assumption of a configuration in which 1) the rotation rate Ω is a function of the cylindrical radius $\varpi = r\cos\theta$ and the concomitant assumption that the pressure P and density ρ can be related by $P = P(\rho)$, i.e. that the configuration is barotropic (even if the underlying equation of state (EOS) is more general (i.e. $P = P(\rho, T)$). (For example, if the configuration is isentropic, or more generally, the temperature T also happens to be a function of ρ in the object.) If these assumptions hold, then the gravitational potential can be algebraically related to the enthalpy $H = \int dP/\rho = H(\rho)$ and the configuration structure can be calculated by an iterative procedure involving the solution of the Poisson equation for the gravitational potential Φ, to which is added the centrifugal potential Φ derived from the rotation curve $\Omega(\varpi)$. Hydrostatic equilibrium yields $H(r,\theta) = \Psi + \Phi$ (up to a constant of integration), which we invert for $\rho(r,\theta) = \rho(H)$ for successive iterations of the object's density structure.

Expansion into Legendre polynomials and radial functions

We choose to follow the method outlined by Boss (1980) and described in Bodenheimer *et al.* 2007. The configuration is expressed in (axisymmetric) spherical polar coordinates (r,θ). The meridional variation of the density and potential is expanded in sums of Legendre polynomials, with radial functions of r for each component, i.e. ρ and potential Ψ:

$$\rho(r,\theta) = \sum_l \rho_l(r) P_l(\cos\theta), \quad \Psi(r,\theta) = \sum_l \Psi_l(r) P_l(\cos\theta).$$

The standard technique of separation of variables applied to the Poisson equation in spherical coordinates leads to an ordinary differential equation for the radial potential function $\Psi_l(r)$ for the component l:

$$\frac{d^2\Psi_l}{dr^2} + \frac{2}{r}\frac{d\Psi_l}{dr} - \frac{l(l+1)}{r^2}\Psi_l = 4\pi G\rho_l. \tag{1}$$

Discretization on a grid r_i yields a tridiagonal matrix system for the discretized version of Ψ_l. If the mass of the configuration is entirely inside the outer boundary r_{out} then the radial equation can be used to give the solution behavior as r tends to infinity. Acceptable solutions behave as $\Psi_l(r) \propto r^{-(l+1)}$, giving a boundary condition at r_{out} of

$$\frac{d\Psi_l}{dr} + \frac{l+1}{r}\Psi_l = 0. \tag{2}$$

For the inner boundary condition $\Psi_l(r=0) = 0$ for $l > 0$, while for $l = 0$, we have boundary condition from the moment equation for ρ, $\Psi_0(r=0) = -4\pi G \int_0^{r_{out}} \rho_0(r) r\, dr$.

Following the solution for Ψ_l we sum over l to find $\Psi(r,\theta)$. From Ψ and the centrifugal potential Φ we find the enthalpy H which is then inverted to get a new estimate for the density ρ. The constant of integration for the enthalpy is set by requiring that the density integrated over the volume equal the desired total mass. The new density estimate is expanded into Legendre components $\rho_l(r)$, and the process is repeated until convergence is achieved.

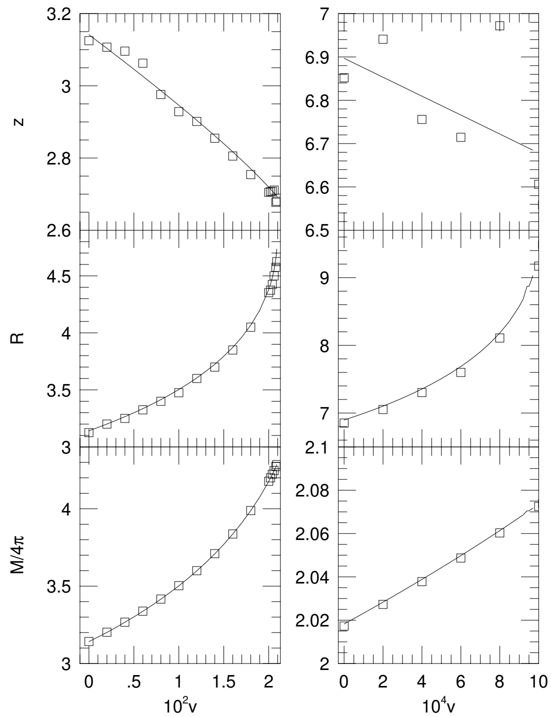

Figure 1: Self-consistent-field (SCF) models of rotating polytropes (indices $n = 1$ and $n = 3$) vs. semi-analytic models computed by R. A. James (1964). From top to bottom, panels show the equatorial radius R, polar radius z and scaled mass $M/4\pi$ for computed models (open squares) versus the James results. Left panels: results for polytropic index $n = 1$. Right panels: results for polytropic index $n = 3$. Legendre components up to $l = 16$ (even components l only) included and radial grid of 200 points. For $n = 1$ the grid outer radius was $r_{out} = 5$ and for $n = 3$ $r_{out} = 10$.

We have developed a program for SCF calculations and have tested it successfully on configurations such as uniformly rotating polytropes (cf. James 1964), for which the pressure is related to the density by a power law $P = K\rho^{1+1/n}$. Results are shown in Fig. 1, which plots the equatorial and polar radii (R and z) and the scaled mass $M/4\pi$ as a function of a measure of the rotation $v = \Omega^2/8\pi G\rho_0(r = 0)$, where Ω is the rotation rate and $\rho_0(r = 0)$ is the density at the origin. Results are shown for polytropic indices $n = 1$ and $n = 3$. For these particular calculations, the angular variation was expanded in Legendre components up to $l = 16$ (even components l only, as equatorial symmetry was assumed) and a radial grid of 200 points was used. For $n = 1$ the grid outer radius was $r_{out} = 5$ and for $n = 3$ $r_{out} = 10$.

Future work will include improvements to take into account issues such as scaling to physical dimensions, realistic equations of state (cf. Korycansky 2019), and non-uniform rotation. Ultimately we hope to create easily-calculable models that can be used to study issues such as the environment of the Earth after the Moon-forming impact.

Acknowledgments

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