

ON THE PROBLEM OF THE FORECASTING FOR METEORITES FALLING. N. I. Perov^{1,2} and Yu. D. Ivanova². ¹State Autonomous Cultural and Education Organization named after V.V. Tereshkova. 150000, Yaroslavl, Ul. Chaikovskogo, 3, Russian Federation. E-mail: perov@yarplaneta.ru. ²State Pedagogical University named after K.D. Ushinskii. 150000, Yaroslavl, Ul. Respublikanskaya, 108, Russian Federation. E-mail: Ivasha_jula@gmail.com.

Introduction: It is considered that the meteorites fall on the surface of the Earth accidentally and unpredictably and the corresponding of fallings places are distributed uniformly on the planet. But in the works [1, 2] these places are distributed far away from the Earth's equator (latitudes φ are greater than $|\varphi| > 30^\circ$). For the interpretation of this phenomenon we pay attention the evolution of the circle orbit of the small body in the frame of a double-averaged Hill's problem with allowance for flattening of the central planet [3, 4, and 5].

The Based Equations in Closed Form: We shall try to determine time P of migration of small body from the Earth's action sphere to the planet and the evolution of the inclination angle i between the planes of the body orbit and the Earth's equator, taking into account the influence of the Sun and the Earth's oblateness. After averaging in Hills' approximation - $a/a_1 \ll 1$ - if the equatorial plane of the Earth and the orbital plane of the star coincides (inclination $i_1=0$) then [3, 4]

$$di/d\tau = -10e^2(1-e^2)^{-1/2} \sin(i)\cos(i)\sin(2\omega). \quad (1)$$

Here i is the inclination, ω is the argument of the perigee. These elements of the osculating body's orbit refer to the plane of perturbed body orbit (and First Point of Aries), e is the eccentricity of the body's orbit. For the considered celestial mechanical problem c_1 , c_2 and a are constants with respect to time (the integrals of the system of the corresponding differential equations [3, 4]). a is the semi major axis of the body's orbit. It should be noted [3, 4]

$$c_1 = (1-e^2)(\cos(i))^2, \quad (2)$$

$$c_2 = \frac{2\gamma}{(1-e^2)^{3/2}} \left(\frac{1}{3} + \cos(2i) \right) + 2(e^2 - (\sin(i))^2) + e^2(\sin(i))^2(5\cos(2\omega) - 3), \quad (3)$$

where

$$\gamma = \frac{1}{2} \frac{\alpha}{\beta}, \quad \alpha = -\frac{3}{8} c_{20} \left(\frac{a_0}{a} \right)^2, \quad \beta = \frac{3}{16} \frac{\mu_1}{\mu} \left(\frac{a}{a_1} \right)^3,$$

$$\tau = \beta(t - t_0) \sqrt{\frac{\mu}{a^3}}.$$

Here μ and μ_1 are the products of the gravitational constant by the masses of the Earth m_E and the Sun m_S respectively; c_{20} is the coefficient of the second zonal

harmonic of gravitational field of the Earth [3, 4]; a_0 is the mean equatorial radius of the Earth; t is Newtonian uniform time; τ is modified time; a_1 is the semi major axis of the circular orbit of the Earth (and the Sun).

From the equations (1), (2), (3) we may find the time P of the close approaching of the small body and the Earth and (or) time of going the body out the Earth's sphere action.

$$P = c_1^2 \cdot \int_{q_{\min}}^{q_{\max}} \frac{dq}{\sqrt{\Psi(q)}}, \quad (4)$$

where

$$\Psi(q) = [8\gamma q^7 - 8/3\gamma q^5 + (4c_1^{5/2} + (-2c_2 + 4)c_1^{3/2})q^2 - 8c_1^{5/2}] \cdot [-8\gamma q^7 + 8/3\gamma q^5 - 20c_1^{3/2}q^4 + (16c_1^{5/2} + 16c_1^{3/2} + 2c_2c_1^{3/2})q^2 - 12c_1^{5/2}]. \quad (5)$$

In equations (4) and (5) we denote: $q = \cos(i)$, $0 < q < 1$; q_{\min} and q_{\max} are positive roots of equation (6)

$$\Psi(q) = 0. \quad (6)$$

$q_{\max} > q_{\min}$, q_{\max} is the nearest root of equation of (6) to q_{\min} . The integral (4) may be used for circulation as well as for libration motion of the perigee.

Examples: Using the equations (1)-(6), we plot the graphs of the functions $i = i(i_0)$, $r_p = r_p(i_0)$, $P = P(i_0)$ (Fig. 1-3). The initial value of the perigee argument of the body's orbit equals $\omega_{in} = 0$ and the final one equals $\omega_{fin} = \pi/2$. In initial moment of time the body moves along quasi-circular orbit, semimajor axis of which is equal $929000 \cdot 10^{3/2}$ m. This value is equal to the half of the Earth's sphere action radius. r_p is the perigee distance of the small body. i_0 and i_f are initial and final inclinations respectively. e_0 and e_f are initial and final orbit eccentricity of the small body respectively. Let us put i_f is a latitude φ of a falling meteorite. It should be noted in this case we have the equality $r_p = a_0$. For the process of migrating of the small body in the gravitational field of the Earth with taking into account the perturbations from the Sun and oblateness of the planet, we put for the considered system (the Earth-the Sun-the small body) $\mu = Gm_E$, $m_E = 6 \cdot 10^{24}$ kg, $a_1 = 1$ AU, $\mu_1 = Gm_S$, $m_S = 2 \cdot 10^{30}$ kg, $a_0 = 6378$ km, $c_{20} = -10^{-3}$. We denote $e_{in} = e_{min} = 0.1$, $i_{in} = i_{max}$, as initial values of eccentricity and inclination of the orbit of the body and $e_{fin} = e_{max}$, $i_{fin} = i_{min}$ are final values; P is the time of the body migration. So, we have from (4) for $e_{in} = 0.1$ and $i_{in} = 82.5918661681^\circ$, $e_{fin} = 0.986273277$, $i_{fin} = 0.6809981$, $rad = 39.0183172^0$, $r_p = 6376.06$ km $\approx a_0$. $P = 2.9955783$ tropical years.

Function $\Psi(q)$ – formula (5) – is shown in Fig. 1 for the given constants. From 14 roots only 2 roots are pointed.

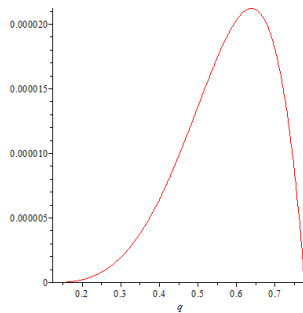


Fig.1. Function $\Psi(q)$. In the given interval of q we have - $q_{\min}=0.12893637$, $q_{\max}=0.7769447301$, $q_m=0.638831101969$ (for the maximum of the function), $q_{inf}=0.499435007642$ (the inflection point of the function).

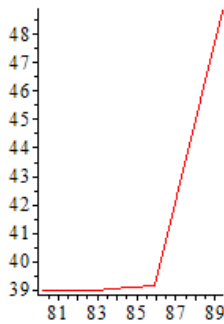


Fig.2. $i_f=i_f(i_0)$. The dependence of r_p upon i_0 is presented in Fig. 3.

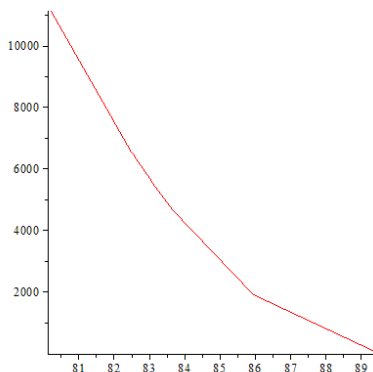


Fig.3. $r_p=r_p(i_0)$. In this model at the small values of $e_{in} \ll 1$ and $i_{in} \ll 90^\circ$ the migrations of bodies from the boundary of the Earth's sphere action into the surface of the planet

are impossible. The last fact follows from the equation (5). (For $e_0 \approx 0$, $q_{in} \approx 1$, $i_{in} \approx 0$ we have $c_1 \approx 1$, $c_2 \approx 0$, $q \approx 1$; $\Psi \approx 0$, if $\gamma \approx 0$). Falling of the meteorites are taking place for the initial inclinations $82.6^\circ < i_0 < 90^\circ$, $39^\circ < \varphi < 90^\circ$ (Fig.2). The dependence of P upon i_0 is presented in Fig. 4.

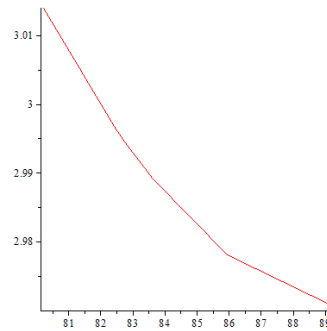


Fig.4. $P=P(i_0)$.
Conclusion:

In this paper we have discussed the problem of the body's orbit evolution and migration of the body from the boundary of the Earth's action sphere to the Earth. The considered celestial mechanical model is based on the double-averaged Hills' problem of secular interactions between a body, and oblate central body (the Earth) and a perturbed body (the Sun).

For the case of coincidence of the equatorial plane of the Earth and an orbital plane of the Sun, determination of period (P) of varying of argument of pericenter of the body's orbit is reduced to a simple quadrature (4) in the evident form *at the first time*. Under the integral sign function $\Psi(q)$ depends upon $q = \cos(i)$ and involves the constants of the first integrals of (2) and (3).

The extreme values of q are determined from the algebraic equation (5) that gives to state the regions of regular varying of the parameters of the orbit of the body as well as to find initials values of Kepler's elements of the body's orbit corresponding to collision of the body with the Earth. In the considered model the time of migration of the body is equal to about 3 years.

Falling of the meteorites are taking place for the initial inclinations $82.6^\circ < i_0 < 90^\circ$, $39^\circ < \varphi < 90^\circ$.

References: [1] https://www.sott.net/image/s6/132156/full/Map_meteors.png. [2] <https://slide-share.ru/image/3724488.jpeg>. [3] Vashkov'jak M.A. (1996) *LAJ*. 22. 231-240. [4] Vashkov'jak M.A. (2018) *S SR*. 52. 69-85. [5] Perov N. I. and Ivanova Yu. D. (2018) *81st Annual Meeting of the Meteoritical Society*. LPI Contribution No. 2067, id.6130.