Scaling laws for stagnant lid convection with buoyant and negatively buoyant crusts

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Introduction

Stagnant lid planets like Venus and Mars are likely to be common among the population of rocky exoplanets. Recent work shows that stagnant lid planets, not just planets with plate tectonics, can potentially be habitable as well. The same processes that regulate climate on Earth, the feedbacks between volcanism and weathering in the carbonate-silicate cycle, can operate on stagnant lid planets through volcanism. Unlike plate tectonic planets, on a stagnant lid planet the crust grows in thickness over time, as there is no recycling via subduction. As the crust grows, it can significantly influence convection in the underlying mantle, and thus also impact the planet's thermal and magmatic evolution. However, a detailed understanding of how stagnant lid convection, in particular the convective heat flux, is influenced by the presence of a thick crust is lacking.

We perform numerical models of stagnant lid convection with a buoyant crustal layer. We find two end-member behaviors: a "**thin crust limit**" where convection is largely unaffected by the presence of the crust, and a "**thick crust limit**" where the crustal thickness itself determines lithospheric thickness and heat flux. We find that the transition between these two regimes occurs when the thickness of the crust plus a sub-crustal thermal boundary layer is approximately equal to the lithosphere thickness convection would produce without a crust. We determine scaling laws for the convective heat flux and mantle internal temperature.

Background

The equations for conservation of mass, momentum, energy, and composition, in terms of non-dimensional variables that the numerical models solve are:

$$\nabla \cdot \underline{v} = 0$$

$$0 = -\nabla P + \nabla \cdot (2\mu \underline{\dot{\varepsilon}}) + Ra_0(T - BC)\hat{\underline{z}}$$

$$\frac{\partial C}{\partial t} + \underline{v} \cdot \nabla C = 0$$

$$\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T = \nabla^2 T$$

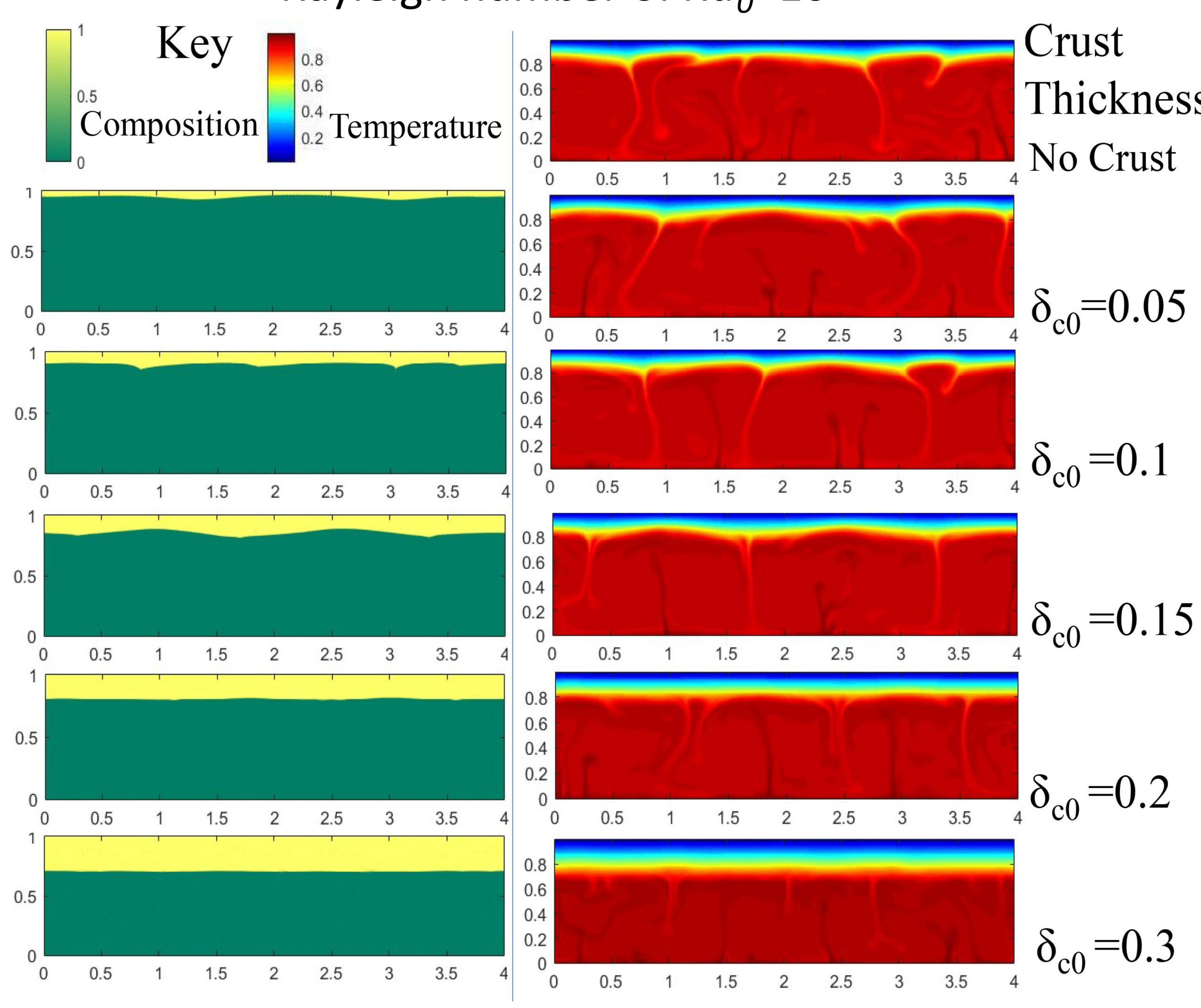
$$Ra_0 = \frac{\rho g \alpha \Delta T d^3}{\kappa \mu_0}$$

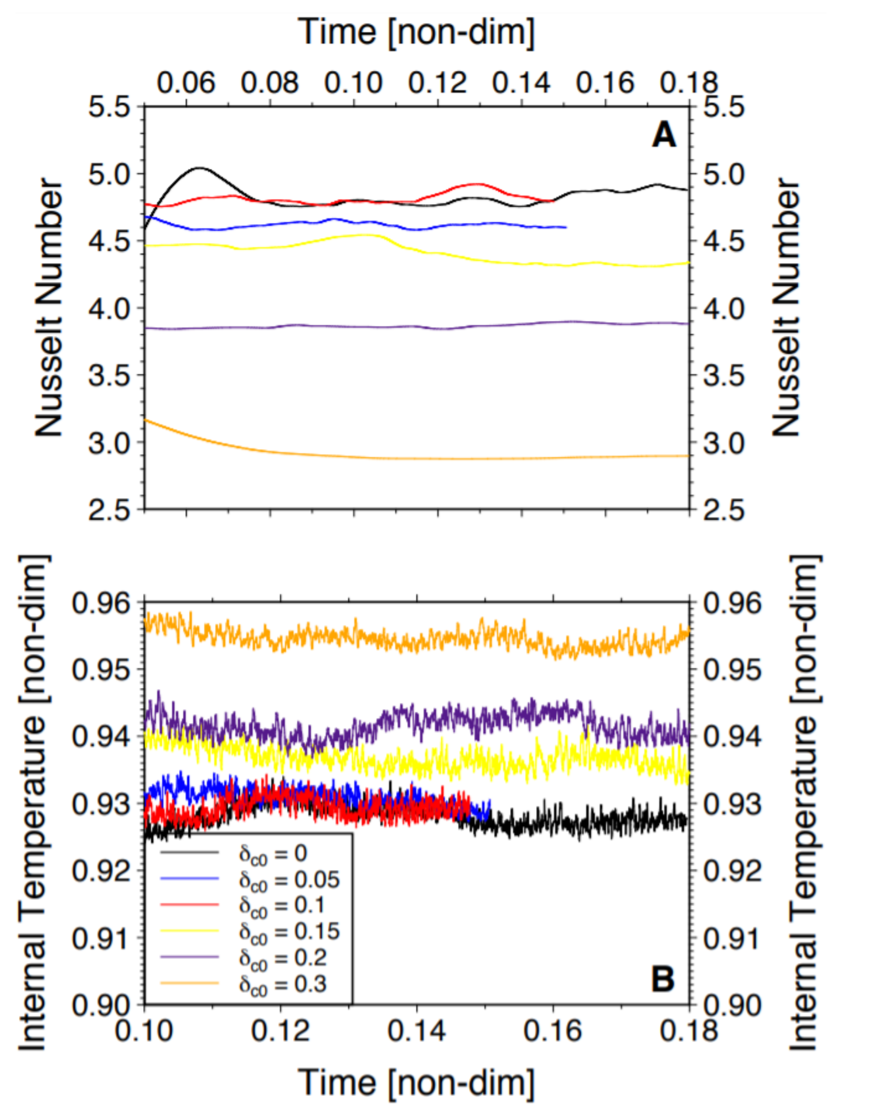
$$\mu = \exp\left(\theta(1 - T)\right)$$

C = Composition; C = 1 is crust, C = 0 is mantle
θ = Frank-Kamenetskii parameter, B = Buoyancy Number, T = Temperature, P = Pressure, $\underline{v}$ = Velocity, μ = Viscosity

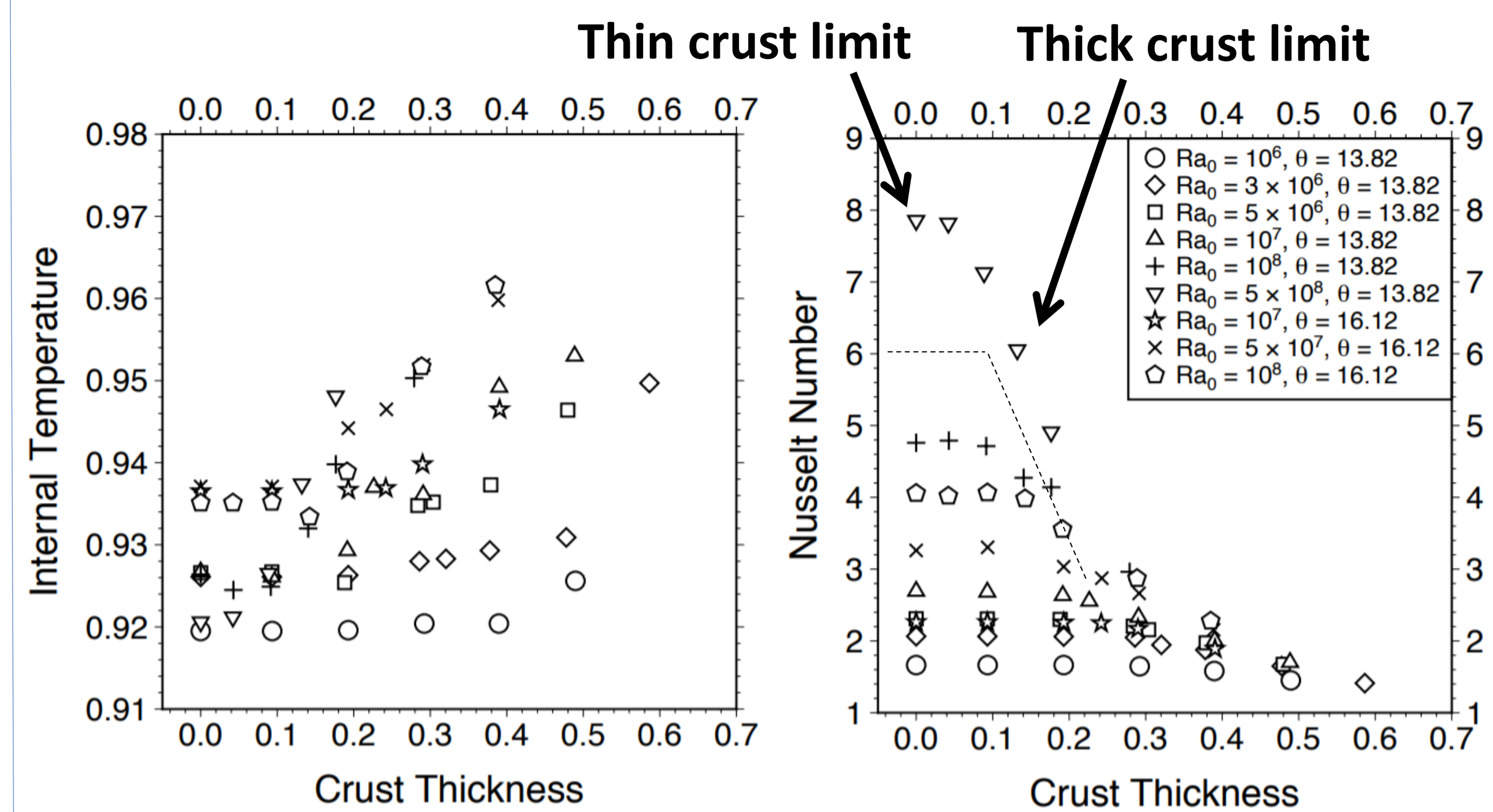
The equations are solved in a 2-D Cartesian domain using a finite volume method. The advection equation for composition is solved using the tracer-ratio method. All models are run in a 4×1 aspect ratio domain with resolution of 512×128. The buoyancy number, B=1 for all models.

Varying Crust Thickness for Rayleigh number of Ra_0=10^8



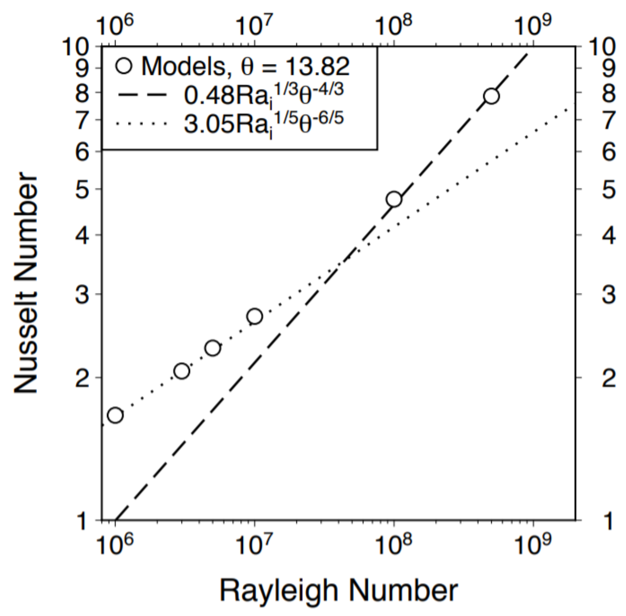


Nusselt Number and Internal Temperature for Ra_0 = 10^8. When the crust is thin, Nusselt number and interior temperature are indistinguishable from the case with no crust. As the crust gets thicker, it begins to dictate the thickness of the lithosphere. As a result, the Nusselt number decreases and internal temperature increases with increasing crust thickness.



Internal temperature versus crust thickness (left) and Nusselt number versus crust thickness (right). Compilation of all model results. There are two clear limiting behaviors. A **"thin crust limit,"** where Nusselt number and interior temperature are largely independent of crustal thickness, and a **"thick crust limit,"** where the thickness of the lithosphere is controlled by the crust thickness. In the thick crust limit a larger crust means a thicker lithosphere, and hence lower Nusselt number and higher interior temperature.

Scaling Analysis: Nusselt number



Models without a crust are well fit by scaling laws for isochemical stagnant lid convection: the steady-state convection scaling law at low Ra_0 and the time-dependent convection scaling law at higher Ra_0. Scaling law proportionality constants are determined by fitting the model results. Models in the **thin crust limit** follow these scaling laws for isochemical stagnant lid convection.
In the thick crust limit crustal thickness determines the lithosphere thickness, such that

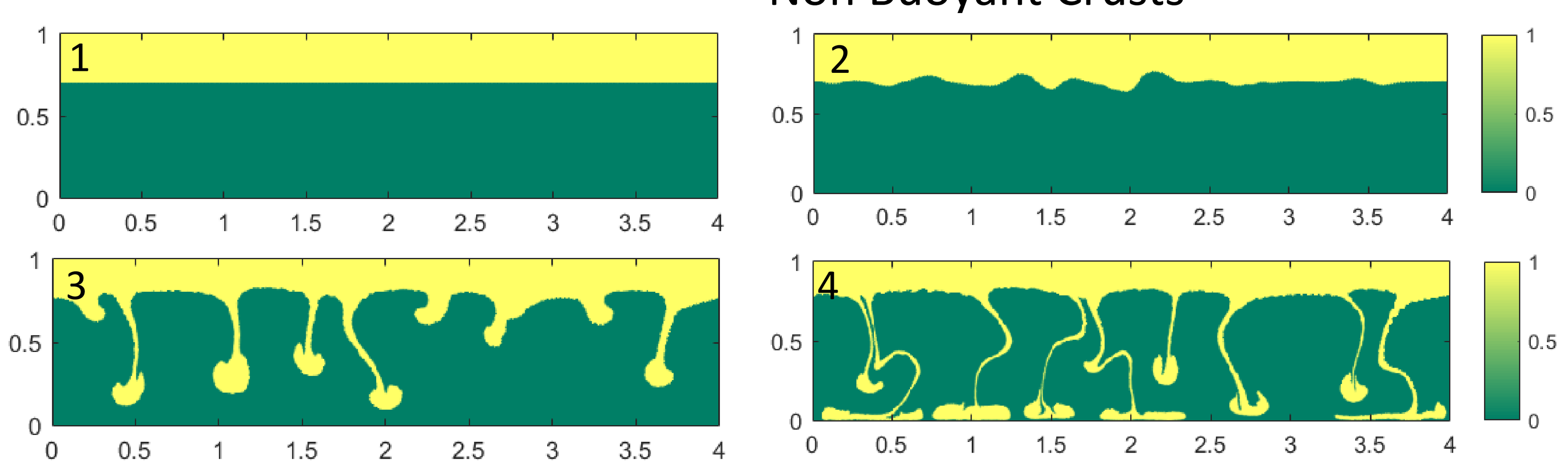
$$Nu = \frac{T_i}{\delta_{sl} + \delta_c} \quad \text{where} \quad \frac{\delta_{sl}^4}{\delta_{sl} + \delta_c} = \frac{Ra_c \mu_i}{Ra_0 T_i}$$

Here δ_c is the crust thickness and δ_{sl} the thickness of lithosphere beneath the crust. Boundary layer stability analysis is used to derive the equation for δ_{sl}.

The complete scaling law combining both limits is:

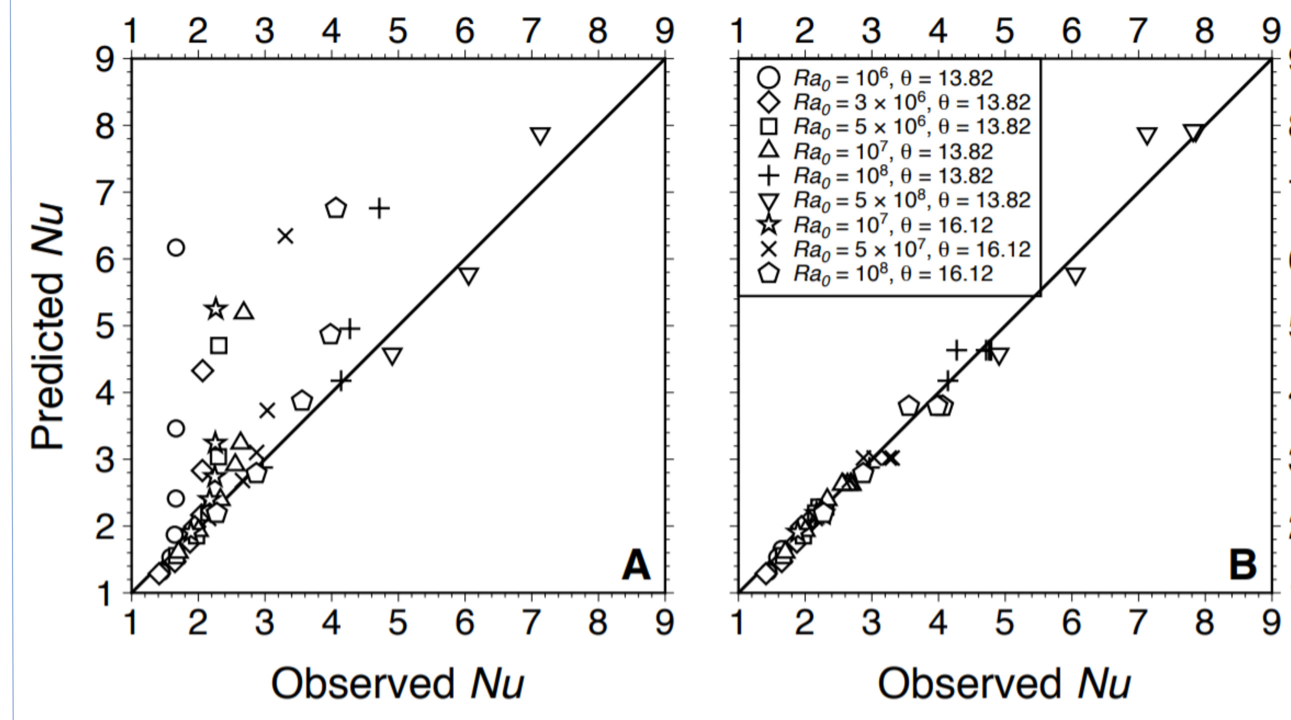
$$Nu = \min\left(\frac{T_i}{\delta_{sl} + \delta_c}, \max\left[C_1\theta^{-\frac{4}{3}}\left(Ra_0\frac{T_i}{\mu_i}\right)^{\frac{1}{3}}, C_2\theta^{-\frac{6}{5}}\left(Ra_0\frac{T_i}{\mu_i}\right)^{\frac{1}{5}}\right]\right)$$

Non Buoyant Crusts



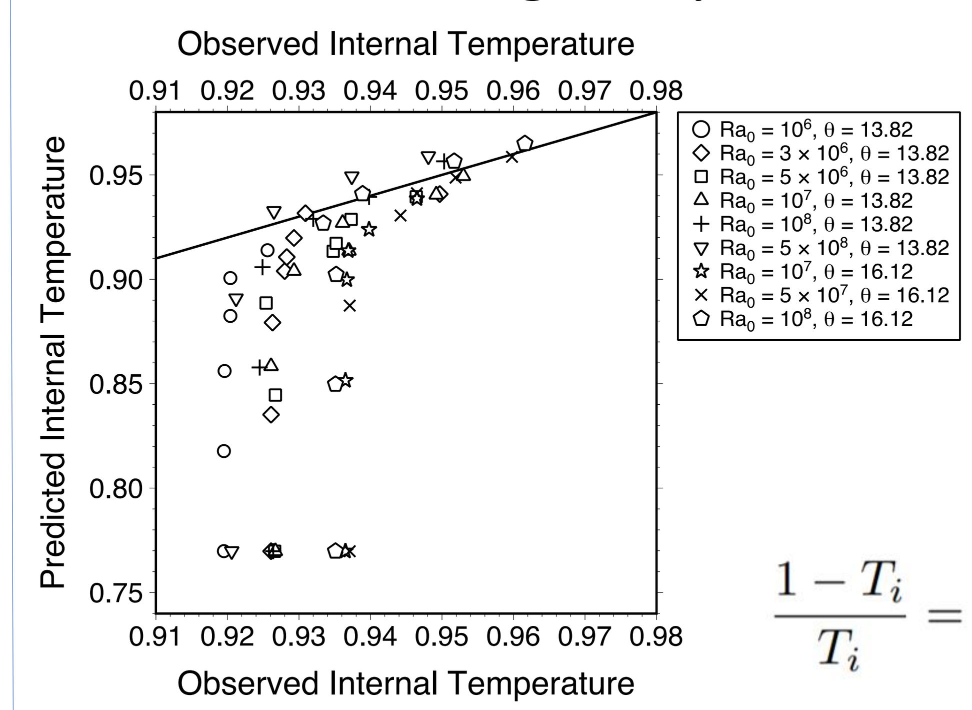
Current work is being performed on the foundering of dense lower crust, in order to determine scaling laws for stagnant lid convection with different crustal densities.

Thick crust limit only vs Full scaling law



Scaling law for Nusselt number fits the numerical model results well, and in particular captures behavior when the crust is thick.

Scaling Analysis: Interior Temperature

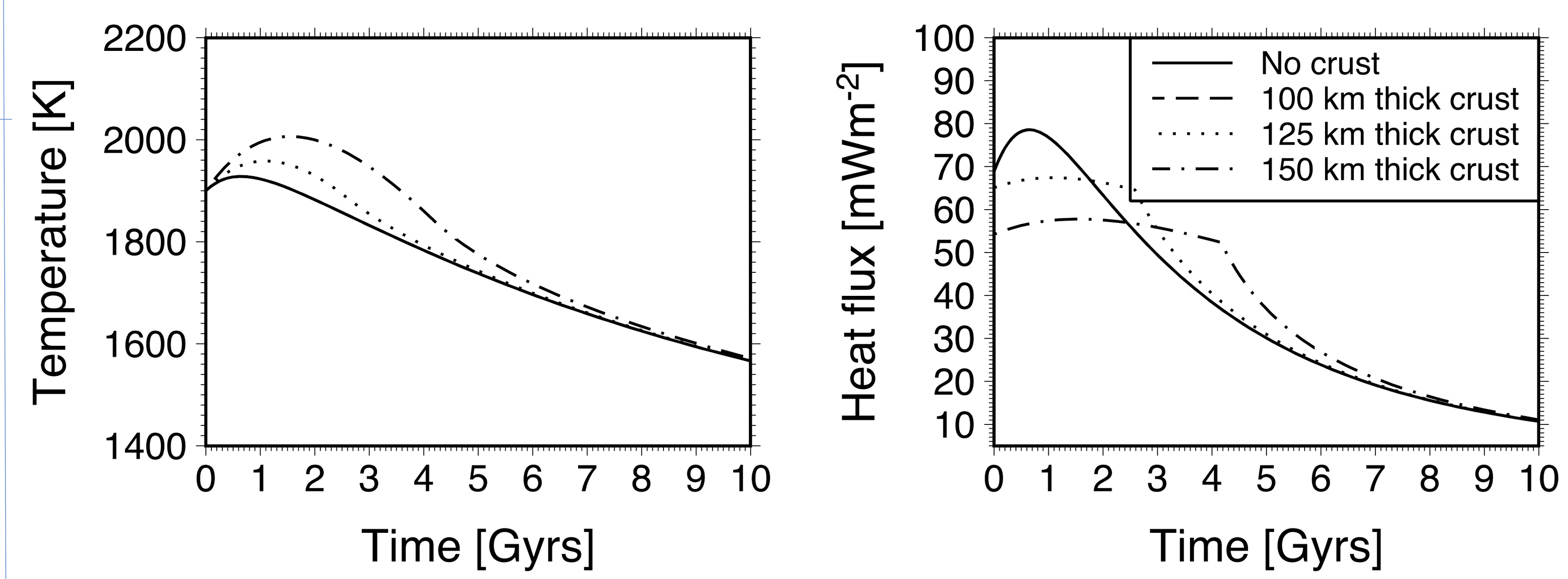


A scaling law for interior temperature in the **thick crust limit** is derived by balancing heat flux at the bottom of the mantle and at the surface. The heat flux at the bottom of the mantle is found using boundary layer stability analysis. This gives the following scaling law:

$$\frac{1 - T_i}{T_i} = \left(\frac{C_3}{\delta_c + \delta_{sl}}\right)\left(\frac{\mu_i}{Ra_0(1 - T_i)}\right)^{\frac{1}{3}}$$

Scaling Law Application: Thermal Evolution Modeling

The presence of a thick, buoyant crust **suppresses mantle heat flux** during early evolution, and causes **mantle temperature to increase**. This early heating phase of evolution can last for billions of years, before evolutionary tracks converge as the mantle cools



Conclusions

-Scaling laws for convective heat flux for stagnant lid convection with a buoyant crust are developed. There are two end member limits: a "thin crust limit" where convection is largely unaffected by the crust, and a "thick crust limit" where the crustal thickness determines the thickness of the lithosphere.

-In the thick crust limit increasing crust thickness leads to a lower heat flux and higher interior temperatures. In a thermal evolution model, a thick crust therefore leads to significant warming of the mantle during the first few billion years of evolution.