ANALYTIC MODEL FOR THE EQUILIBRIUM TEMPERATURE DISTRIBUTION OF A SUNLIT GAUSSIAN AIRLESS SURFACE L. Rubanenko[1], N. Schörghofer[2] and D. A. Paige[1] [1]University of California, Los Angeles, CA, USA, [2]Planetary Science Institute, Tucson, AZ, USA (liorr@ucla.edu)

Introduction: Insolation dominates the heat balance on sunlit airless surfaces compared to scattered and emitted radiation from nearby slopes and subsurface heat diffusion [1]. Surface slopes affect the temperature distribution by decreasing the downward component of the flux vector and affects the directional scattering in high phase angles. In order to account for these effects, thermophysical models often include an illumination component in which scattering and shadowing are calculated using computationally extensive techniques such as ray-tracing [2, 3, 4]. Here we derive an analytic thermophysical rough surface model that assumes normally distributed slopes and Lambert scattering that calculates the surface temperature distribution from its slope distribution

Our statistical roughness model: A Gaussian surface is a common way to model roughness statistically. Elevation and the slope in each direction are distributed Gaussian and uncorrelated with one another [5, 6, 3, 1]. Each slope on the surface is described by the bidirectional slope vector $\vec{s}$ whose components (p, q), the directional slopes, are normally distributed [5]. The slope magnitude is calculated as $|\vec{s}| \equiv s = \sqrt{p^2 + q^2} = \tan\alpha$, where α is the slope angle. The compass direction the slope is facing is the slope aspect $\theta = \arctan q/p$, defined as the angle between the projection of the slope normal on the reference plane and the positive direction of the y-axis (north) measured clockwise. Since p, q are normally distributed, we may trivially find the distribution of the slope angle α,

$$f_\alpha(\alpha) = \frac{\tan\alpha}{\omega^2\cos^2\alpha}\exp\left(-\frac{\tan^2\alpha}{2\omega^2}\right) \quad (1)$$

where ω is the root mean square (RMS) slope.

Linking Roughness and Temperature: The solar radiation reaching a sunlit slope depends on the solar incidence angle Θ who is defined with respect to the slope normal vector,

$$\cos\Theta = \cos z \cos\alpha + \sin z \sin\alpha \cos(\theta - a_s) \quad (2)$$

where z is the solar zenith angle, a_s is the solar azimuth angle and θ is the slope aspect. The flux reaching each slope on the surface depends on the cosine of the incidence angle and the distance to the sun r,

$$F = \frac{S_0\,(1-A)}{(r/1\,\mathrm{AU})^2}\cos\Theta \equiv \beta\cos\Theta \quad (3)$$

where $S_0 = 1367\ \mathrm{Wm}^{-2}$ is the average solar constant at 1AU and A is the surface albedo. Finally, the equilibrium temperature may be derived from the incident flux assuming the surface is a black-body with emissivity ε,

$$F = \sigma\varepsilon T^4 \quad (4)$$

where $\sigma = 5.67\times10^{-8}\ \mathrm{W\,m^{-2}K^{-4}}$ is the Stefan-Boltzman constant. Throughout this work, we assume radiative equilibrium, $A = 0.1$ and $\varepsilon = 0.95$ as representative values for the bond albedo and emissivity of the lunar surface.

Symbol	Definition
α	The slope angle
p, q	The directional slopes
ω	The root mean square slope
Θ	Slope incidence angle
z	Solar zenith angle
a_s	Solar azimuth angle
S_0	The solar constant at 1 AU

Table 1: Symbols used in this work and their definitions.

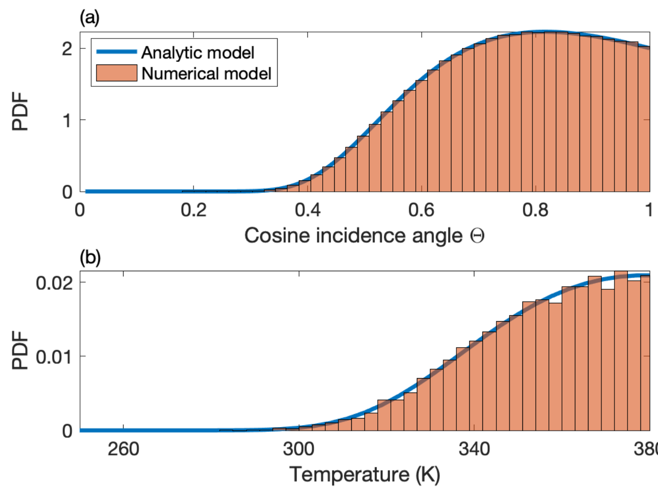

Figure 1: The incidence angle distribution (Panel a) and equilibrium temperature distribution (Panel b) for a rough surface with a Gaussian slope distribution, with RMS slope angle $45°$ and zero zenith angle. The distributions computed by our analytic model (blue line) match the numerically modeled distribution (orange bars).

Special case: Sun in zenith: At zenith, the solar incidence angle equals the slope angle (Eq. 2), and the distribution of incidence angles is the same as in Eq. 1 with $\Theta = \alpha$. Using change of variables, we derive the flux distribution and the resulting temperature distribution for a rough surface when the Sun is in zenith,

$$f_T(T) = \frac{4}{\omega^2\rho^2T^9}\exp\left(-\frac{1}{2\omega^2}\frac{1-\rho^2T^8}{\rho^2T^8}\right) \quad (5)$$

In Figure 1 we validate our model with a numerical illumination model [4] by simulating a rough random surface with RMS slope angle $45°$.

We additionally derive a useful expression for the distribution of emitted flux; the flux distribution F_e as seen by an observer found at some angle γ measured from the zenith. Assuming small slope angles, this distribution is given by,

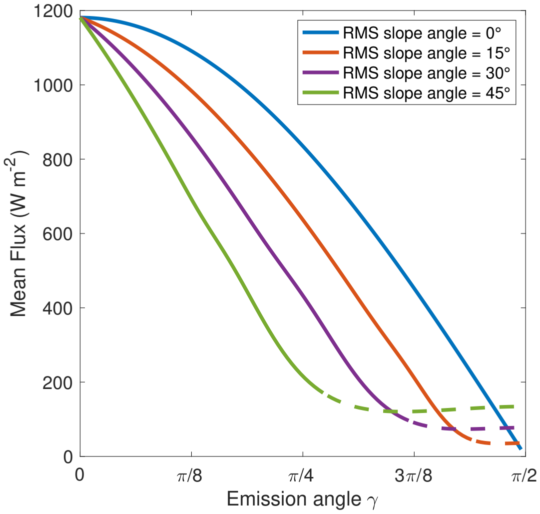

Figure 2: The average flux emitted from a rough surface depends on the observation angle γ measured from zenith (Eq. 7). Since we do not account for mutual shadowing, the flux slightly increases for highly oblique observation angles. The range in which this becomes important is indicated by the dashed line. For higher RMS slope the derivative near 0 is not zero due to our small angle approximation.

$$f_{F_e} = \frac{F_e - \beta \cos\gamma}{\omega^2 \beta^2 \sin^2\gamma} \exp\left(-\frac{(F_e - \beta\cos\gamma)^2}{2\omega^2\beta^2\sin^2\gamma}\right) \quad (6)$$

The expected value of this flux is given by,

$$\bar{F_e} = \sqrt{2}\omega\beta\sin\gamma \cdot \left[\Gamma\left(\frac{3}{2}, \frac{\cot^2\gamma}{2\omega^2}\right) - \frac{\sqrt{\pi}}{2}\right] - \beta\cos\gamma \cdot \left[\Gamma\left(1, \frac{\cot^2\gamma}{2\omega^2}\right) - 1\right] \quad (7)$$

Figure 2 shows the average flux reaching an observer found at an observation angle γ from zenith. Per our assumption of Lambertian scattering, the observed decrease in flux for a flat surface is given the cosine of the observation angle. Additionally, since we do not account for mutual obscurations, the mean flux increases for oblique observation angles. This model artifact is indicated by the dashed lines in Figure 7. In the future, we hope to overcome this by accounting for mutual shadowing.

General case: For zenith angles $z \neq 0$, the solar azimuth term in Eq. 2 does not cancel out. Consequently, calculating the incidence angle distribution is no longer trivial, and solving an integral with no closed-form solution,

$$I = \int_a^b \frac{\left[(x-a)(b-x)\right]^{-\frac{1}{2}}}{x^3} e^{-\frac{1}{2\omega^2 x^2}} dx \quad (8)$$

where $a = \cos(\Theta - z)$ and $b = \cos(\Theta + z)$. Here we solve it numerically, hoping to obtain a series solution in the future. An analytic series solution will not only surpass extensive numerical models in computation speed, but will also allow us to further investigate our parameter space. Figure 3 shows the incidence angle probability density function for various RMS slope angles and $z = 25°$. For flatter surfaces (lower RMS slope), the incidence angle distribution converges to the solar zenith angle, as expected.

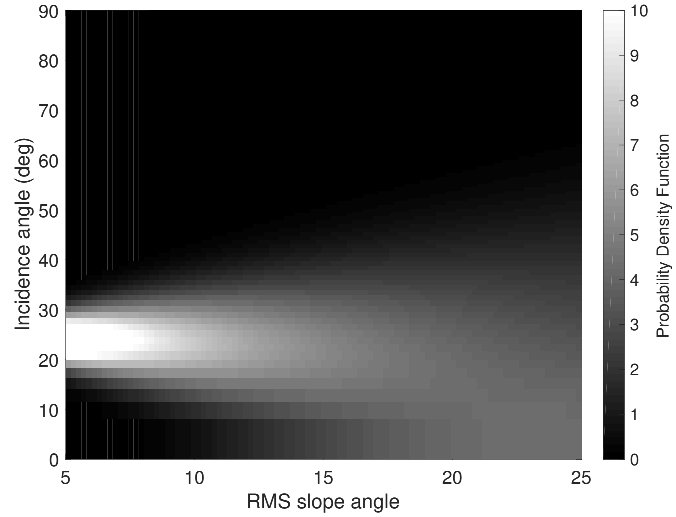

Figure 3: The 2-D probability density function for different RMS slope angles and incidence angles and $z = 25°$. For flatter surfaces (lower RMS slope), the incidence angle distribution converges to the solar zenith angle.

References

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