Using effective density spectrum to constrain crustal density profile of Vesta and Ceres. Anton I. Ermakov[1], Ryan S. Park[1], Carol A. Raymond[1], Julie C. Castillo-Rogez[1], Christopher T. Russell[2]. [1]Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, 91109, USA (anton.ermakov@jpl.nasa.gov); [2]University of California Los Angeles, IGPP/EPSS, Los Angeles, CA, 90095, USA.

Introduction:

The gravity field of Vesta has been determined by the Dawn mission up to spherical harmonic degree 18 [1]. The gravity field of Vesta is strongly correlated with the gravity derived from Vesta's shape [2]. The latest gravity model of Ceres is globally accurate up to degree and order 14 [3], which is sufficient to resolve the gravitational signatures of several largest impact basins [4]. While Vesta's gravity indicated that its topography is uncompensated [2], observed gravity anomalies and gravity topography admittance spectrum reveal that Ceres is globally close to isostatic equilibrium [3-6], despite having deviations from isostasy on regional scales. Put in a geological and geomorphological context, the regional deviations from isostasy give clues on the structure and evolution of Ceres' crust.

The goal of this paper is to present a methodology to constrain the vertical density profile from the gravity and topography data that would be applicable for irregularly shaped bodies. The vertical density profile is an important aspect of crustal structure as it depends both on chemical and physical stratification of the body.

Effective density spectra of Vesta and Ceres:

The effective density is defined as simply the ratio of the gravitational potential spherical harmonic coefficients to the gravity-from-shape spherical harmonic coefficients multiplied by the mean density of the body [e.g. 7]. However, for bodies where the correlation between gravity and gravity-from-shape is close to unity, it is more convenient to define the effective density spectrum as:

$$\tilde{\rho}_n = \sqrt{\frac{V_n}{V_n^{\mathrm{const}}}}\,\bar{\rho},$$

where V_n is the variance of the gravitational potential spherical harmonic coefficients:

$$V_n = \sum_{m=0}^{n}(\bar{C}_{nm}^2 + \bar{S}_{nm}^2).$$

V_n^{const} is the variance of the gravitational potential spherical harmonic coefficients of the body assuming it is homogeneous in density; *n* is degree and *m* is order. Since higher-degree spherical harmonic coefficients sample shallower structures and density is typically increasing with depth, the effective density spectrum usually decreases with *n*.

The observed effective density spectra for Vesta and Ceres are shown in Fig. 1. The spikes at low even degrees are caused by the high hydrostatic flattening observed at both bodies. The effective density spectrum of Vesta generally decreases from 2>*n*>18, whereas that of Ceres increases. Ceres' topography does not viscously relax for scales less than 250 km [4], which corresponds to a spherical harmonic degree of 11. At higher degrees, the gravity-topography admittance is expected to not be influenced by the relief of the crust-mantle boundary (i.e. the compensating relief) but rather by the vertical density structure of the crust. As we will show below, we can use the effective density spectrum to constrain the vertical density profile.

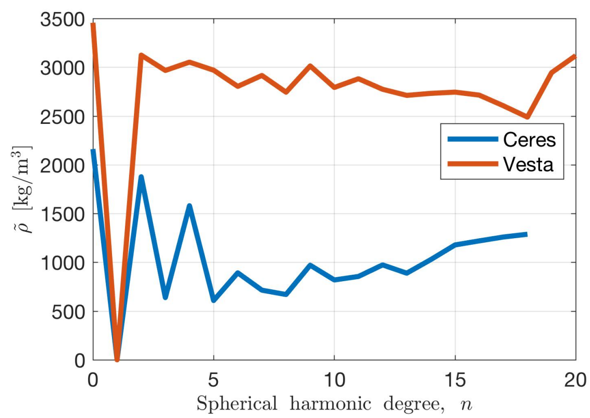

Fig. 1: Effective density spectra of Vesta and Ceres.

Derivation of the effective density spectrum:

We first define a short notation for the normalized spherical harmonic gravity coefficients:

$$\bar{C}_{nm} = \sigma_{1nm};\ \bar{S}_{nm} = \sigma_{2nm},$$

Similarly to [7], we assume that the body consists of multiple layers, each of which has a constant density. Also, we assume that the shape of these layers is the downscaled version of the outer shape of the body. This assumption effectively limits our analysis to the parts of the effective density spectrum that are not affected by isostatic compensation, since the bottom boundary of the isostatically compensated layer is a mirror image of its top boundary (i.e. outer surface).

The gravity coefficients of the outer shape are $\sigma_{inm}^{\mathrm{const}}$ and they are referenced to the volumetric radius of the body *R*. The gravity coefficients of the inner layers are also $\sigma_{inm}^{\mathrm{const}}$ if they are referenced to the corresponding volumetric radii, but their contribution to the overall gravity σ_{inm} needs to be weighted by the upward propagation factor $(r_i/R)^n$. In addition, $\sigma_{inm}^{\mathrm{const}}$ needs to be weighted by the fractional mass of the layers. In summary, σ_{inm} takes the following form:

$$\sigma_{inm} = \frac{1}{M}\left(\sigma_{inm}^{\text{const}}\frac{4}{3}\pi r_1^3\rho_1\left(\frac{r_1}{R}\right)^n + \sigma_{inm}^{\text{const}}\frac{4}{3}\pi r_2^3(\rho_2-\rho_1)\left(\frac{r_2}{R}\right)^n + \cdots + \sigma_{inm}^{\text{const}}\frac{4}{3}\pi r_k^3(\rho_k-\rho_{k-1})\left(\frac{r_k}{R}\right)^n\right),$$

where r_i are the volumetric radii of layers ($r_1 = R$, i.e. the volumetric radius of the outermost layer) and ρ_i are the densities of the layers. Taking $M=4/3\pi R^3\bar{\rho}$, where M is the total mass and $\bar{\rho}$ is the mean density, we get after simplification:

$$\sigma_{inm} = \frac{\sigma_{inm}^{\text{const}}}{\bar{\rho}R^{n+3}}\left(r_1^{n+3}\rho_1 + r_2^{n+3}(\rho_2-\rho_1) + \cdots + r_k^{n+3}(\rho_k-\rho_{k-1})\right),$$

Now, we are replacing the sum by an integral. The density difference between layers is, therefore, also replaced by a differential expression: $(\rho_k - \rho_{k-1}) \to -\frac{d\rho(r)}{dr}\cdot dr$. We get the following integral for the gravity coefficients:

$$\sigma_{inm} = \frac{\sigma_{inm}^{\text{const}}}{\bar{\rho}R^{n+3}}\int_{R^+}^{0} r^{n+3}\cdot\frac{d\rho(r)}{dr}\cdot dr,$$

where R^+ signifies that integration needs to go past the outer interface. The effective density spectrum is defined as:

$$\tilde{\rho}_n = \frac{\sigma_{inm}}{\sigma_{inm}^{\text{const}}}\bar{\rho}.$$

Therefore, we get an integral expression for $\tilde{\rho}_n$

$$\tilde{\rho}_n = \frac{1}{R^{n+3}}\int_{R^+}^{0} r^{n+3}\cdot\frac{d\rho(r)}{dr}\cdot dr.$$

At the outer interface $r=R$, the density goes instantaneously from surface density ρ_{surf} to zero. Therefore, the derivative of the density with respect to volumetric radius has a singularity at $r=R$. Replacing the value of the derivative at $r=R$ by a Dirac delta function times the amplitude of the density jump: $\rho_{\text{surf}}\cdot\delta(r-R)$, we can integrate through $r=R$ and get:

$$\tilde{\rho}_n = \rho_{\text{surf}} + \frac{1}{R^{n+3}}\int_{R^-}^{0} r^{n+3}\cdot\frac{d\rho(r)}{dr}\cdot dr.$$

$\tilde{\rho}_n$ has the following two interesting properties:

1) At n=0, $\tilde{\rho}_0 = \bar{\rho}$. Therefore, when fitting the modelled effective density spectrum to the observed one, we can add an extra observation for n=0.
2) For $n \to \infty, \tilde{\rho}_\infty = \rho_{\text{surf}}$. Therefore, the high-degree effective density spectrum approaches the surface density of the body.

Density profiles:

We can use the derived expression for $\tilde{\rho}_n$ with simple density profiles. The two-layer density model results in the following effective density spectrum:

$$\tilde{\rho}_n = \rho_{\text{upper}} + \left(\frac{r_{\text{lower}}}{r_{\text{upper}}}\right)^{n+3}(\rho_{\text{upper}} - \rho_{\text{lower}})$$

A linear density profile results in the following density spectrum:

$$\tilde{\rho}_n = \frac{n\rho_{\text{surf}} + 4\bar{\rho}}{n+4},$$

Or, alternatively, if expressed in terms of the central density:

$$\tilde{\rho}_n = \frac{4n\bar{\rho} + 12\bar{\rho} - n\rho_{\text{center}}}{3(n+4)}.$$

If we adopt an exponential density profile with an offset:

$$\rho(r) = \rho_0 + \Delta\rho e^{\frac{r-R}{d}},$$

the effective density spectrum takes the following form:

$$\tilde{\rho}_n = \rho_{\text{surf}} + (-1)^n\left(\frac{d}{R}\right)^{n+3}\Delta\rho e^{-R/d}[\Gamma(n+4) - \Gamma(n+4, -R/d)],$$

where $\rho_{\text{surf}} = \rho_0 - \Delta\rho$, $\Gamma(z)$ is the gamma function and $\Gamma(a,z)$ is the upper incomplete gamma function.

We note that unlike [7], we did not use the mass-sheet approximation in deriving the effective density spectra. Therefore, the derived formulas can be used for constraining the density profiles of irregularly shaped asteroids given their observed effective density spectra.

Future work with Dawn second extended mission data:

The Dawn spacecraft was in highly elliptical low altitude-pericenter orbit around Ceres from June 6, 2018 until it lost its attitude control capability on Oct 31, 2018. Due to the nature of the orbit, useful high-resolution gravity data was collected only near the pericenter. The high-resolution gravity data covers a narrow swath approximately 2/3 of a great circle length that crosses Occator and Urvara Craters, two important Ceres landmarks. Preliminary analysis shows that the Dawn measurements in this narrow swath are sensitive to gravity up to $n = 40 - 45$. Therefore, we plan to conduct a localized gravity-topography analysis of these data and derive local estimates of the effective density spectrum, which, in turn, will be used to constrain vertical density profile to much shallower depths compared to the primary mission data.

References:

[1] Konopliv et al., (2014) *Icarus*, 240, 103-117; [2] Ermakov et al., (2014) *Icarus*, 240, 146-160; [3] Konopliv et al., (2019) *Icarus*, 299, 411-429 [4] Ermakov et al., (2017) *JGR:Planets*, 122 (11), 2267-2293; [5] Park et al., (2016) *Nature* 537,515–517; [6] Fu et al., (2017) *EPSL*, 476, 153-164; [7] Besserer et al., (2014) *GRL,* 41.16: 5771-5777.