PLANET-PLANET TIDAL HEATING IN THE TRAPPIST-1 SYSTEM Hamish C. F. C. Hay\textsuperscript{1} and Isamu Matsuyama\textsuperscript{1, 2} Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ 85721, United States (hhay@lpl.arizona.edu)

Introduction: In the TRAPPIST-1 system [e.g., 1] the shortest conjunction distance between the two inner planets is less than two times the Earth-Moon separation distance. In contrast, the maximum distance between these planets is ten times our Moon’s semimajor axis. Each TRAPPIST-1 planet is subject to a highly time-varying tidal forcing from its neighbors due to these extreme variations in separation distance. We investigate this phenomenon by developing the first theory to account for these planet-forced tides and subsequent heating in compact planetary systems like the TRAPPIST-1 system and Galilean satellites.

This phenomenon was first noted in [2, 3], where the time-varying tidal forcing due to neighboring planets was compared to the other source of periodic tides: the planet’s eccentric orbit. Planet-forced tides differ from the eccentricity tide because they can cause time-varying distortion and heating even when the perturbed body has a circular orbit. To determine if the effect of planet-forced tides is significant, it must be compared to these eccentricity-forced tides from the central body. It is not sufficient, though, to compare the magnitude of these two types of tidal forcing because the eccentricity-forcing operates at the orbital frequency, whereas the planet-forcing operates at the conjunction frequency. This is important because the response of any planet or moon to a periodic forcing is dependent on the forcing’s frequency. In this work we treat these planets as homogeneous Maxwell materials to account for this frequency dependence.

Theory: Determining the magnitude and spatial distribution of tidal forces across a planet’s surface relies on two fundamental unknowns, the direction, \(\hat{\phi}_{ij}\), and distance, \(|\vec{r}_{ij}|\), to the tide-raising body (Figure 1). Here the \(ij\) subscript refers to an inner planet \(i\) and outer planet \(j\). We calculate these two unknowns as a function of time using each planet’s star-planet separation vector, \(\vec{r}_i\) and \(\vec{r}_j\). We assume that the system is coplanar and the orbits are circular.

Using these assumptions, we can then calculate the degree-2 planet-forced tidal potential through [4];

\[
\Phi^P(\theta, \phi, t) = \frac{G m_j}{r_{ij}} \left( \frac{R_i}{r_{ij}} \right)^2 \left( \frac{3 \cos^2 \gamma - 1}{2} \right)
\]

(1)

where \(m_j\) is the outer planet’s mass, \(R_i\) is the inner planet’s radius, \(G\) is the universal gravitational constant, and \(\gamma\) is given by,

\[
\cos \gamma = \sin \theta \cos(\phi - \phi_{ij})
\]

(2)

Figure 1: Diagram of the problem. Two planets \(i\) and \(j\) orbit around a central body with separation vectors \(\vec{r}_i\) and \(\vec{r}_j\), respectively.

where \(\theta\) and \(\phi\) are the inner planets’ colatitude and longitude, respectively, assuming zero obliquity. Additionally, we assume that the planet is synchronously rotating.

Figure 2 shows the tidal potential at the substellar point on planet-g as a function of time for eccentricity and planet forcing. While the eccentricity-forced potential is a simple periodic function, the planet-forced po-

Figure 2: Tidal potential at the substellar point \((\theta = \pi/2, \phi = 0)\) on planet-g in the TRAPPIST-1 system. Eccentricity (dashed) and planet-forced (blue) tides were calculated using the masses, radii and eccentricities from [5] and [6]. The shaded region shows the 1-sigma uncertainty in eccentricity.
potential is a complex waveform composed of a sum of sinusoidal functions at different frequencies, $q$. In order to investigate tidal heating from planet-forcing, we must take into account each of these frequencies separately.

We calculate the spherical harmonic expansion coefficients of the tidal potential (Eq. 1) as a function of time, and then further expand those into a set of Fourier series coefficients using a discrete Fourier transform. The normalised frequency spectrum of the spherical harmonic degree $l = 2$ and order $m = 2$ Fourier coefficient for each planet is shown in Figure 3. Most strikingly, the planet forcing does not peak at the conjunction frequency, but rather at higher frequencies. This is very different to the eccentricity-forced tidal heating. Additionally, we investigate moon-moon tides between the Galilean satellites to estimate the magnitude of the resulting tidal displacements.

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**References:**