SMALL BODIES' STRENGTH: FAILURE MODEL. S. Voropaev¹, Y. Jianguo² and J-P. Barriot³,
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Introduction: It was assumed that barrage of comets and asteroids produced many young lunar basins (craters over 300 kilometers in diameter) during an impact spike at ~3.7 Ga (Late Heavy Bombardment, LHB). Many authors assume the LHB ended about 3.7 to 3.8 billion years (Gyr) ago with the formation of Orientale basin, but recent analysis of the possible ⁴₀Ar diffusion in Apollo samples make this assumption questionable [BP]. Evidence for LHB sized blasts on Earth extend into the Archaean and early Proterozoic eons, in the form of global ejecta layers of impact spherule beds. At least seven spherule beds, formed between 3.23 and 3.47 Gyr ago, have been found, four between 2.49 and 2.63 Gyr ago, and one between 1.7 and 2.1 Gyr ago [SM].

Recently, in the geological time scale, late Eocene impact clusters were formed: the Mt Ashmore structural dome at Timor sea (D ~ 90 km, 35.4 ± 0.1 Ma), Popigai in East Siberia (D ~ 100 km, 35.7 ± 0.2 Ma) and the Chesapeake Bay in North America (D ~ 85 km, 35.3 ± 0.1 Ma). The formation event could have been a rupture of a huge asteroid with a size of 30-40 km into pieces under 10 km. New data on the east Antarctic gravity field from the Gravity Recovery and Climate Experiment (GRACE) mission revealed a prominent positive free-air gravity anomaly over ca. 500 km diameter subglacial basin centered on (70° S, 120° E) in north central Wilkes Land [RF]. The inferred impact crater is nearly three times the size of the Chicxulub crater (D ~ 180 km, 65 ± 0.2 Ma) and presumably formed at the beginning of the greatest extinction of life on Earth at ca. 260 Ma. The Siberian Traps were antipodal to it.

Potential sources of such uninvited guests are near-Earth asteroids families – Apollons, Atons, etc. Their debris regularly cross the Earth's orbit and, once in a hundred years, the fall of a body with size ~10 m occurs. The probability of significantly larger falling fragments, at a size of ~1 km, is much smaller, with a recurrence of a few million years. Nevertheless, disastrous consequences for the entire Earth’s biosphere makes detailed studies of their strength and rate of decay under the action of tidal forces an active subject.

Failure model: At present, the geological composition of small bodies of the Solar system is better known from a number of spacecraft missions to asteroids and comets (Hayabusa, NEAR-Shoemaker, Rosetta etc.). They are mainly consolidated bodies, made of rocks, metal compounds and ice, with dramatic surface structures full of large craters and/or system of cracks which should disappear in the case of full fragmentation and subsequent re-accumulation. A detailed study of the meteorite Chelyabinsk (LL5 S3 W0 ordinary chondrite, fall 15 February 2013) revealed that its parent body’s composition was breccias of pristine chondrite material and crystalized impact melts with hard coupling with each other. One of its main rest' fragment is shown on Figure 1 and thoroughly done measurements provided the mechanical strength of underlying surface rocks [SV].

![Fig. 1. Meteorite Chelyabinsk’ LL5 fragment: (a) – pristine ordinary chondrite (b) – impact melt. Breccias size ~ 35x25x15 cm, mass ~ 4 kg, mean density ~ 3.2 g/cm³.](image)

Apparently, melting and subsequent solidification are common processes following shock wave propagation on asteroids and comets after impact events. Therefore, to consider small bodies’ failure states of stress, we need a failure model, as used in rock mechanics.

The Grady & Kipp [GK] dynamic fragmentation model treats the fracturing material as if it were a continuum. The average effect of many individual fractures is incorporated into a scalar parameter called damage, D. The parameter D is defined such that D = 0 corresponds to intact, undamaged rock, and D = 1 means the material is completely fragmented and unable to transmit tensile stress - i.e. the body is a collection of separate fragments. In this model, the yield strength, Y, is given by

$$Y = (1 - D) Y_i + D Y_d,$$  \hspace{1cm} (1)

where Yᵢ and Yᵩ are the yield strengths of intact and shock-damaged rock, respectively. The yield strength
\( Y_d \) is well established and is known as the Coulomb friction law,
\[
Y_d = Y_{coh} + \mu_{dam} P,
\]
where \( Y_{coh}, \mu_{dam} \) and \( P \) are the cohesion, internal friction and pressure, respectively. The \( Y_d \) is limited by the von Mises plastic limit, which is typically 1-5 GPa under strong compression. The maximum value of \( \mu_{dam} \sim 0.6 \) is typical for rocky granular media.

Recent observations have indicated that rubble pile asteroids (\( D = 1 \)) may have a small, but finite, level of tensile strength allowing them to have rotation rates above their spin deformation limit [HS]. Estimation of maximum possible tensile strength provides only value \(-10 \text{ KPa}\). Popova et al. [PO] assembled data on 13 cases of meteoritic falls and estimated the bulk strength of the objects corresponding to their earliest observed fragmentation in the high atmosphere. In all 13 cases, the strengths were much less than the compressive or tensile strength reported for that class of stony meteorites. Bulk compressive strengths upon atmospheric entry of these bodies were shown to be very low: 0.1 – 1 MPa on first breakups and 1-10 MPa on final breakups.

From the [PO] we can see that pre-entry, 10-100 meter scale interplanetary meteoroids are typically highly fractured (0.5 < \( D < 0.8 \)) and can break up under tensile stresses of 0.03 + 1 MPa. The exception is the fall (September 15, 2007) of the Carancas stony meteorite at Peru which caused the formation of a 13 m wide impact crater. It was classified as an ordinary chondrite H4-5 with an estimated initial size \( \sim 0.9 \) + 1.7 m, a compressive strength \( \sim 20 + 40 \) MPa and a tensile strength \( \sim 1.2 + 2.4 \) MPa, depending of the trajectory. So, the Carancas meteorite is a rare example of a monolithic meteoroid that was almost free of internal cracks (0 < \( D < 0.2 \)).

**Results:** One of the most important questions in this context is at what distance from a large planet such a body may split, and how this distance depends on the physical properties of the body. This has been under debate ever since Edouard Roche, calculating his famous expression for the splitting distance, \( D \), as
\[
D = 2.45 (\rho_r/\rho) \sqrt{R_p}\text{ or } \delta = D/R_p (\rho_r/\rho)^{1/3} = 2.45,
\]
where \( R_p \) and \( \rho_p \) are the radius and density of the planet, and \( \rho_r \) is the density of the small body. Roche used a homogeneous, self-gravitating liquid satellite on a circular orbit around a solid planet, no reference to any other property of the body than its density.

Let us consider the stability of a rotating prolate body during a close encounter with Earth, in a general scaling form. As we know from the detailed analysis [VJ], the tensile (positive) stress, \( \sigma_{xx} \), appears first on the surface equator at the point \( x = z = 0, y = c, \) for some distance \( D_c \) ("down" case). The parameter \( D_c \) could be defined as
\[
\sigma_{xx}(D, \nu, \varepsilon) = I_c/Z(k,t) \left[ (1/\delta^3)T(k,t) + 3*(I_c/l_s)^2R(k,t) - 6*(f_0(\varepsilon)*F_0(k,t) + f_2(\varepsilon)*F_2(k,t)) \right],
\]
where \( k(\nu) = (1 - \nu)/(1 - 2v), t = 1 - \varepsilon^2 = c^2/a^2, \nu \) is the Poisson’ ratio, as usual (see [SV] for details about functions \( T(k,t), \) etc.). Therefore, \( \delta_c \) is defined by the condition \( \sigma_{xx}(D_c, \nu, \varepsilon) = 0\):
\[
(\delta_c)^3 = T(k,t) / 3*[2*(f_0(\varepsilon)*F_0(k,t) + f_2(\varepsilon)*F_2(k,t) - (I_c/l_s)^2R(k,t)]
\]
This scaling form, connecting the dimensionless variables \( \delta = D/R_p (\rho_r/\rho)^{1/3} \) and \( \chi = L_s/R_p = \rho_r/\rho_0, \rho_0 = \pi/G*\tau^2 \), allows to analyze the critical distance for any possible combination of physical parameters of asteroid and planet. For a sphere \( \varepsilon = 0, \delta_c \) takes the simplest form
\[
(\delta_c)^3 = 5*(1 - \nu^2) / [(7 + 5\nu^2)*(1 - 2v) - \chi^2*6^*(4 - 3\nu^2)]
\]
On Figure 2 (a, b) different \( \delta_c \) cases are presented for the stony and icy prolate bodies, respectively.

![Fig. 2. Critical distance, \( \delta_c \), in dimensionless form for different composition of asteroids and rotation: (a) stony rock, \( \nu = 0.2, \rho_r = 2.67 \text{ g/sm}^3 \); red line – \( T = 3 \) hours (h), blue dotted – \( T = 5 \) h, green dashed – \( T = 0 \) h; (b) mixed ice, \( \nu = 0.3, \rho_r = 0.86 \text{ g/sm}^3 \), red line – \( T = 5 \) hours (h), blue dotted – \( T = 6 \) h, green dashed – \( T = 0 \) h. The horizontal pointed dashed line corresponds to 2.45 – the Roche limit for a liquid body with a tensile strength equal to zero.](image)