OPTIMAL SURFACE RELIEF RETRIEVAL FROM THE SET OF PHOTOMETRIC AND ALTIMETER DATA.

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Introduction: The interpretation of available by the time images of planetary surfaces taken during space missions often requires knowledge about surface topography. The procedure for recalculation of the image's brightness into surface heights was proposed by Van Diggelen [1]. This procedure is based on a priori known dependence of the surface facet brightness on its orientation. The method is still widely used (for example, [2, 3]), though as shown in [4] in its initial formulation [1] it seems to be a mathematically incorrectly posed problem.

In this work we present the method for the surface topography retrieval based on both images and heights data recorded by an altimeter with a wide radiation pattern. This approach allows the most probable surface height variations consistent with source data.

Heights from Images: To solve the announced problem we used "photometric" approach. Observed images' brightness has to be recalculated into the slopes field $\mathbf{t}(x,y)$ using a priori known brightness-tilts relation. The gradient of true surface heights $H(x,y)$ is different from calculated slopes due to a random registration noise $\delta(x,y)$:

$$\nabla H(x,y) = \mathbf{t}(x,y) - \delta(x,y). \qquad (1)$$

The presence of a random noise is a reason for the problem formulated in [1] to be a mathematically incorrectly posed. In general, due to $\delta(x,y)$ (1) cannot be solved with probability of 1. Instead, we use the Bayesian statistical approach [5] to find the most probable statistical estimation of true heights.

We consider the true relief and the image noise to be realizations of stationary Gaussian processes with spectral densities $I_H(k_x,k_y)$ and $I_\delta(k_x,k_y)$, respectively. k_x and k_y are spatial frequency vector $\mathbf{k}$ components.

Wide Radiation Pattern Altimeter: Other set of data $h(x,y)$ involved into the problem under study is observations made by an altimeter having a wide radiation pattern $D(x,y)$:

$$h(x,y) = \int D(x - x',y - y')H(x',y')dx'dy' + \varepsilon(x,y), \quad (2)$$

where $\varepsilon(x,y)$ is a random noise of altimeter data.

Similar to (1), if there is no noise, $\varepsilon(x,y) = 0$ and (2) can be considered as an integral equation which determines the true relief. But, if $\varepsilon(x,y) \neq 0$, the solution of (2) can be only found in the frame of the statistical approach as the most probable statistical estimation for true heights based on $h(x,y)$. In such a case we consider $\varepsilon(x,y)$ to be a realization of the stationary Gaussian processes with spectral density of $I_\varepsilon(k_x,k_y)$.

The statistical estimation is based on the Bayesian equation [5] which gives the relation between event's a posteriori and a priori probability densities. Fourier components of the stationary Gaussian process are statistically independent to each other. Thus a heights Fourier -component estimation can be obtained for every (k_x,k_y). The use of both images and altimeter observations leads to the following result:

$$\tilde{H} = \frac{\tilde{D}^*\tilde{h}/I_\varepsilon - i\mathbf{k}\tilde{\mathbf{t}}/I_\delta}{1/I_H + \tilde{D}^*\tilde{D}/I_\varepsilon + \mathbf{k}^2/I_\delta}. \qquad (3)$$

Wave sings denote Fourier transforms of corresponding variables.

The inverse Fourier transform of results obtained through (3) gives the most probable relief of the surface under study according to data used.

When information about slopes $\mathbf{t}(x,y)$ is absent, the spectral density $I_\delta(k_x,k_y)$ is infinite and expression (3) is transformed into an usual Wiener filter based on altimetry measurements $h(x,y)$ only:

$$\tilde{H} = \frac{\tilde{D}^*\tilde{H}}{\tilde{D}^*\tilde{D} + I_\varepsilon / I_H}. \qquad (4)$$

And vise versa, if there is no altimetry information, which means that $I_\varepsilon(k_x,k_y) = \infty$, the expression (3) is transformed into

$$\tilde{H} = \frac{-i\mathbf{k}\tilde{\mathbf{t}}}{\mathbf{k}^2 + I_\delta/I_H}. \qquad (5)$$

The accuracy of calculated heights depends on images and altimeter data noise levels. Spatial resolution of the most probable topography is limited by the resolution of source images.

Any number of images can be used for calculations of the most probable surface relief. Actually, the use only one image also allows calculation of the most probable relief [6], but at least two images with different solar azimuths are needed to estimate both components of the heights gradient. Solar azimuths normal to each other are the most preferable observational conditions.

Test Calculations: We modeled a lunar-like cratered surface to test the approach discussed above. The model relief is shown in Fig. 1a. An average value of heights was chosen to be zero; their root-mean-square value was σ_0.

The relief (Fig. 1a) was used to model altimeter observations. Gaussian function with a blur radius of 32 resolution elements was considered to be an altimeter radiation pattern. Also we add a random noise to model altimeter data, their signal-to-noise ratio ($\mathrm{SNR_A}$) was chosen to be 1, 10, 100, and 1000. An example of such an observation with SNR_A = 10 is shown in Fig. 1b.

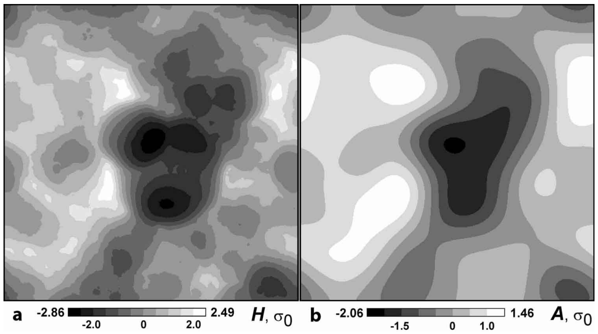

Fig. 1. The test relief: (a) the initial relief, heights vary from -2.36σ_0 to +2.21σ_0; (b) the altimeter observation with SNR_A = 10, heights vary from -1.91σ_0 to +1.5σ_0. Darker shades correspond to lower heights.

The same test relief (Fig. 1a) was used to model pairs of optical images having normal to each other solar azimuths and SNR_I equal to 1, 10, 100, and 1000. We assumed the Lambert's law as an a priori known photometric function of the surface. For these experiments, surface albedo was considered to be the same over the surface under study. The pair of optical images with SNR_I = 10 is presented in Fig. 2.

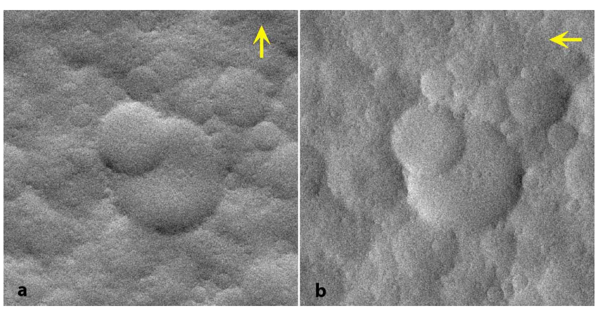

Fig. 2. Initial images with SNR_I = 10. Directions of illuminations are shown with arrows.

The most probable surface relief was calculated using both optical images and altimeter observations having different combination of SNR_I and SNR_A. In all cases the retrieved surface relief is very similar to the model one.

Retrieved heights deviations from the model relief for various SNR_I and SNR_A combinations are presented in Table 1. It is obvious that the use of altimeter data allows decreasing the deviations of the most probable relief from the model one up to an order of magnitude.

Spatial distributions of absolute differences between the calculated relief and the model one are shown in Fig. 3 for several SNR_I and SNR_A pairs.

Conclusions: The photometric method is the most mathematically rigorous. It is based on the statistical approach and allows calculation of the most probable surface heights distribution consistent with source images and altimeter data.

Table 1. The mean-square retrieved heights deviations from the model relief in terms of σ_0.

SNR_A	SNR_I			
	1	10	100	1000
1	0.088	0.038	0.016	0.007
10	0.081	0.032	0.013	0.005
100	0.070	0.028	0.010	0.004
1000	0.063	0.023	0.008	0.003

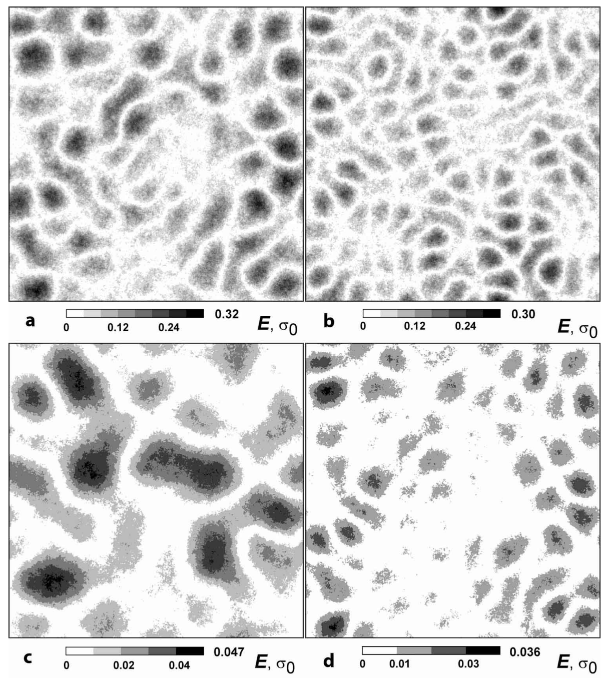

Fig. 3. Distributions of absolute differences between the retrieved relief and the model one calculated with photometric method using images and altimeter data: (a) SNR_I = 1, SNR_A = 1; (b) SNR_I = 1, SNR_A = 100; (c) SNR_I = 100, SNR_A = 1; (d) SNR_I = 100, SNR_A = 100. Darker shades correspond to lower deviations values.

References: [1] Van Diggelen J. (1951) *Neth. Astron. Inst. Bull., 11*, 283-289. [2] Dulova I. A. et al. (2008) *Telecom. Rad. Eng., 67*, 1605-1620. [3] Lohse V. et al. (2006) *PSS*, *54*, 661-674. [4] Robinson M. S et al. (2010) *Space Sci. Rev., 150*, 81-124. [5] Bayes, T. (1958) *Biometrica, 45*, 293-315. [6] Dulova I. A. et al. (2008) *Sol. Sys. Res.*, *42*, 522-535.