The temperature distribution of a sunlit lunar surface: an analytic model

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Introduction

During the day, insolation dominates the surface heat balance on slowly rotating airless planetary bodies such as the Moon. Shadowing, scattering and thermal emission from topography become gradually important as the solar incidence angle increases. Thusfar, models calculating the daytime temperatures of the lunar surface utilized computationally expensive techniques [1] or incorporated statistics with simplifying assumptions [2,3]. Here we adopt a probabilistic approach to obtain a closed-form solution for the temperature distribution of sunlit rough surfaces and explore the full parameter space of the problem.

Methods

We begin by assuming a surface with Gaussian slope distribution and, by change of variables, derive the temperature distribution it would have if placed on the Moon. The 2-D slope magnitude distribution of a Gaussian random surface is given by,

$$P_s\left(s\right) = \frac{s}{\omega^2}\exp\left(-\frac{s^2}{2\omega^2}\right)$$

where s is the tangent of the slope angle α and ω is the root mean square (RMS) slope angle. For each slope on the surface, the local solar incidence angle is given by,

$$\cos\Theta = \cos\alpha\cos z + \sin\alpha\sin z\cos(\theta - a_s) \qquad (1)$$

where z is the solar zenith angle (incidence angle with respect to a flat surface), a_s is the solar azimuth, α and θ are the slope angle and aspect. Using change of variables, we may obtain the slope incidence angle probability density function, and then, similarly, the solar flux and temperature distribution.

Results

For the simple case of $z = 0$ (sun in zenith), the second term in (1) vanishes and the probability function can easily be integrated to give the surface temperature distribution,

$$f_T(T) = \frac{4}{\omega^2\rho^2T^9}\exp\left(-\frac{1}{2\omega^2}\frac{1-\rho^2T^8}{\rho^2T^8}\right); \quad (\text{for z} = 0) \qquad (2)$$

We can use this result to calculate the mean flux emitted by the surface, assuming radiative equilibrium,

$$\bar{F}_e = \beta\left[1 - \Gamma\left(1, \frac{1}{2\omega^2}\right)\right]$$

Where $\beta = S_0\left(1-A\right)/(r/1\,\mathrm{AU})^2$; $\rho \equiv \sigma\varepsilon/\beta$, A and ε are the bolometric albedo and emissivity, σ=5.67x10^{-8} is the Stefan-Boltzmann constant, and r the distance from the Sun. Expanding this in ω $(= \beta[1-\omega^2 + O(\omega^4)])$ shows a fraction ω^2 of the total incident energy is scattered between surface slopes.

For the general case (*i.e.*, any z) the second term in (1) no longer vanishes and the integral generating the probability density function no longer has an analytic solution. However, an approximation for this integral may be obtained in cases where $|\cos(\Theta - z)| \ll 1$ (the local and global incidence angles are significantly different),

$$f_T = \frac{4\omega}{\sqrt{2\pi}}\frac{\chi}{T}\sqrt{\frac{\zeta}{1-\chi^2}}\left(1+\frac{1}{\omega^2 f}\right)\exp\left(\frac{1}{2\omega^2}\left(1-\frac{1}{f}\right)\right) \qquad (3)$$

$$\chi = \sigma\varepsilon/\beta;\ \zeta = 1 + \frac{\chi\cot z}{\sqrt{1-\chi^2}};\ f = \left(1-\chi^2\right)\zeta^2\sin^2 z$$

Figure 2 shows a comparison between our analytic model and a computationally extensive temperature model [4].

Future work

Next we plan to generalize our model for other incidence and solar zenith angles. Possible uses for our model include predicting surface temperatures measured by radiometers such as the Diviner Lunar radiometer experiment [5] in emission angles other than zero. Lastly, we intend to include scattering and shadowing, potentially by including previous probabilistic models such as Smith (1967) [6].

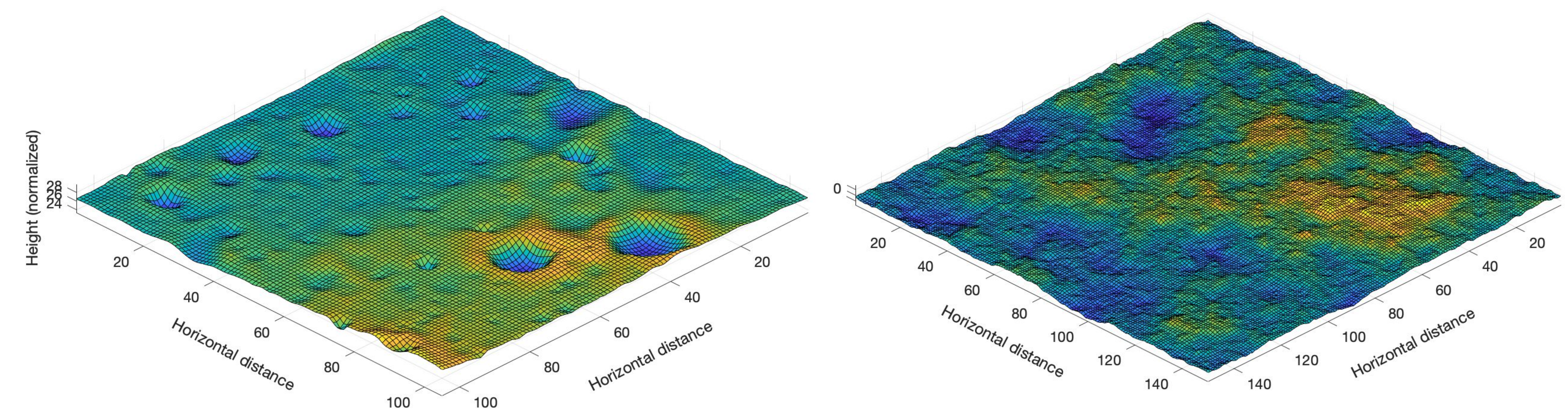

Figure 1: a graphical representation of a rough random Gaussian surface, compared to realistic lunar topography sampled by the Lunar reconnaissance Laser Altimeter (LOLA [7]). Realistic topography is characterized by the presence of impact craters, whose slope distribution has a longer tail compared to a Gaussian distribution. Colormap indicates elevation.

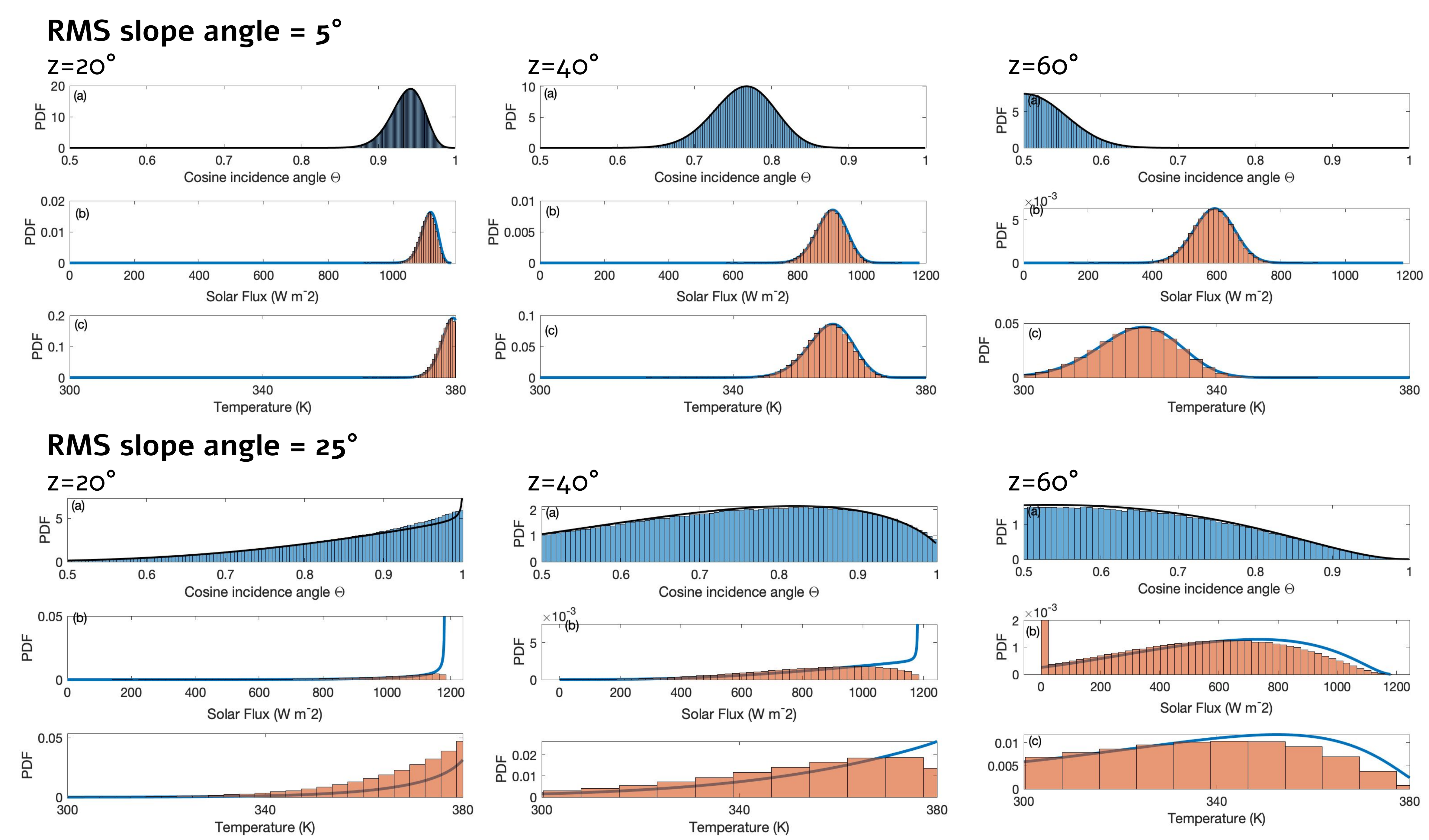

Figure 2: some comparisons between our analytic model (lines) and the temperature distribution of a rough Gaussian surface calculated by a computationally expensive thermal model [4] (bars). Our analytic results agree with the numerical model for low RMS slope angles and significantly different local and global solar zenith angles. z is the solar zenith angle, the incidence angle with respect to a flat surface.

References

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