# Evection Resonance in the Earth-Moon System

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## Introduction

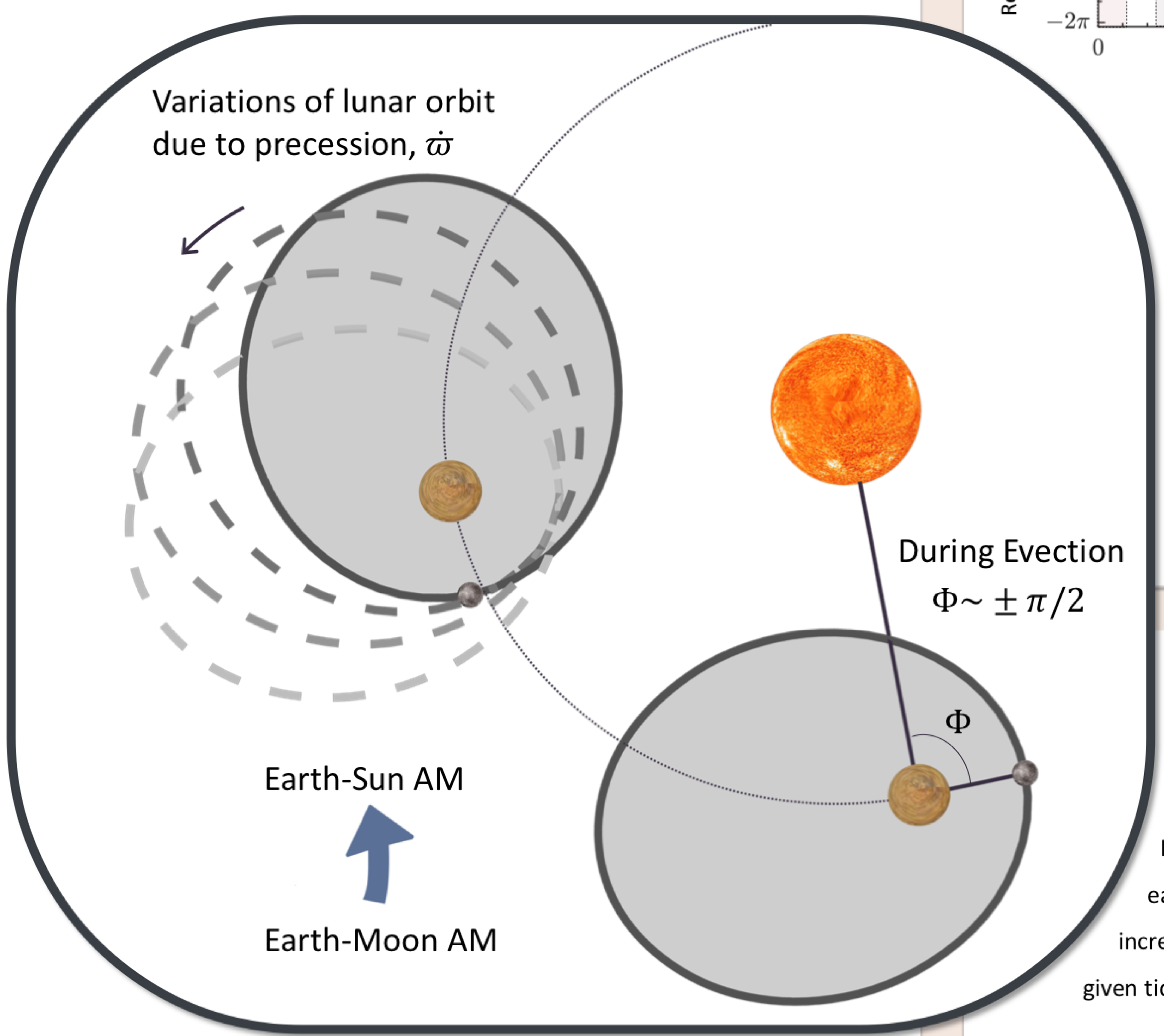
Moon formation by a high-**angular momentum** (AM) impact may offer a compelling mechanism to create a satellite that is compositionally similar to the silicate Earth [1, 2, 3]. In such impacts, the Earth-Moon system's initially high AM (i.e. $L > 2L_{EM}$) must be greatly reduced after the Moon forms. A possible AM removal mechanism is the evection resonance with the Sun [1]. Evection occurs when:

$$\text{Lunar Precession rate } \dot{\varpi} = 2\pi/1\,\text{yr} \text{ Earth's Orbital rate}$$

Evection removes AM from the Earth-Moon system, transferring it to the Earth's orbit. For an initial 5-hr terrestrial spin (corresponding to a total AM of $\sim 1\,L_{EM}$, the current Earth-Moon AM), the resonance is encountered at 4.6 Earth radii ($R_{\oplus}$) and only limited AM removal was found [4]. However, with a **more rapidly spinning Earth** (total AM $L_{\text{tot}} > 2L_{EM}$), the resonance location shifts outward due to Earth's increased oblateness, and large-scale AM removal was found [1]. Notably, there also appeared to be a preference for a final AM near $L_F \sim 1\,L_{EM}$, independent of the starting AM. However, follow-up studies have found contradicting outcomes (e.g., early resonance escape), and limited AM removal efficiency in evection proper.

To explore the origin of such differences and to assess the robustness of evection for removing AM from the Earth-Moon, we study the system's early evolution throughout evection assuming a large range of possible tidal parameters.

### How much angular momentum is removed by evection?




*Not to Scale

## Methods

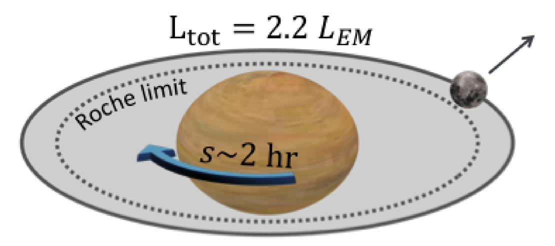
We assume Earth is on circular heliocentric orbit, and a non-inclined lunar orbit. We evolve the Moon's **semimajor axis** ($a$), **eccentricity** ($e$), **Earth and Moon spin rates**, and the **evection resonance angle** ($\Phi$, measuring the difference between the solar longitude and the Moon's perigee position as seen from Earth) using the **Mignard tidal model** [5]. The tidal model assumes a constant time lag, $\Delta t$, between the tide-raising-potential and the body's response, which can be related to a tidal dissipation factor:

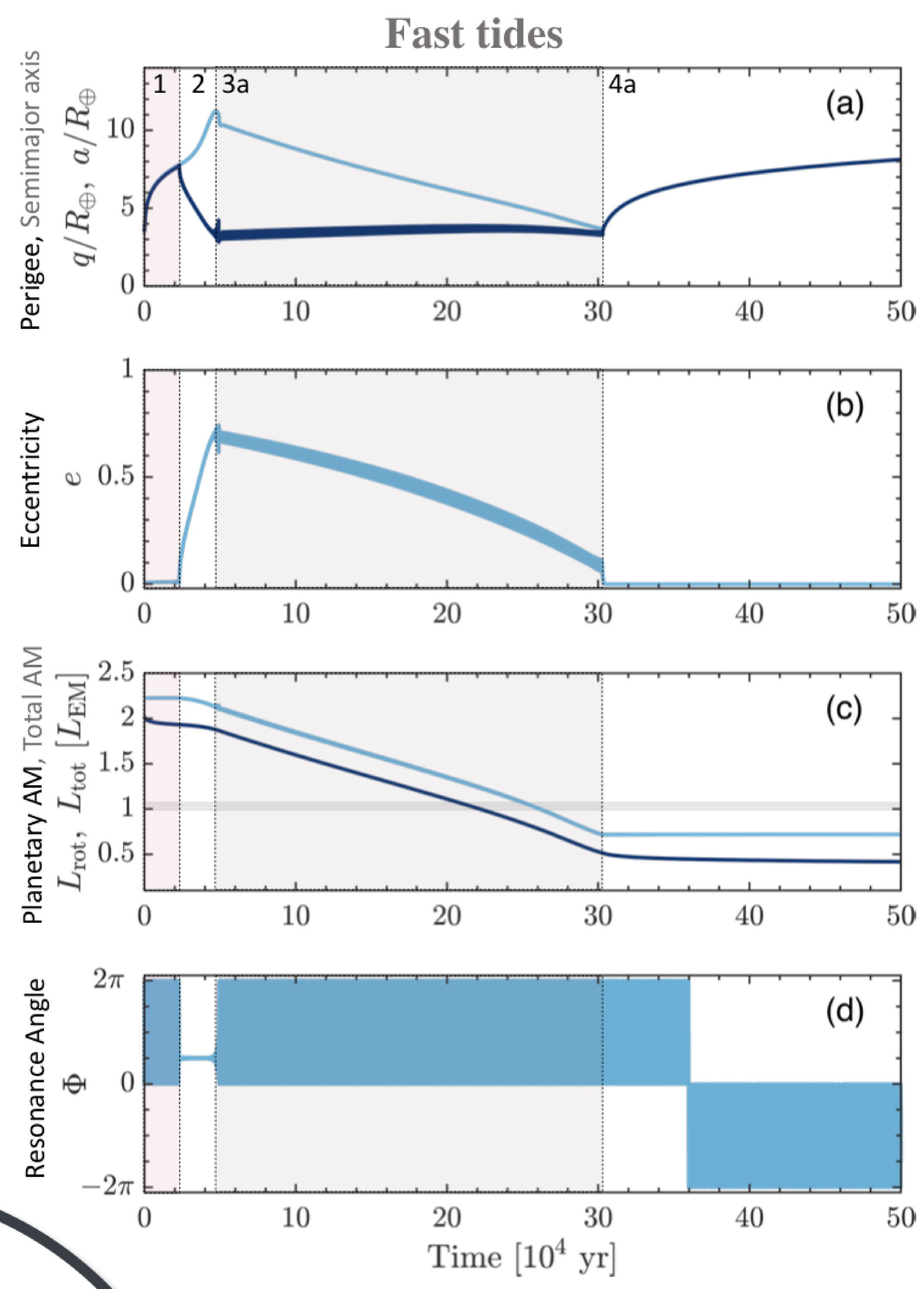
$$Q_{\text{eff}} \sim [2(s-n)\Delta t]^{-1}$$

where $s$ is the Earth's spin rate and $n$ is the Moon's mean motion.

The simulations start with a Moon outside the Roche limit on a near-circular orbit around a fast-rotating Earth (2 hr, $L_{\text{rot}} = 2L_{EM}$), with an initial total AM of $L_{\text{tot}} = 2.2\,L_{EM}$.

## Two Evolutionary Regimes



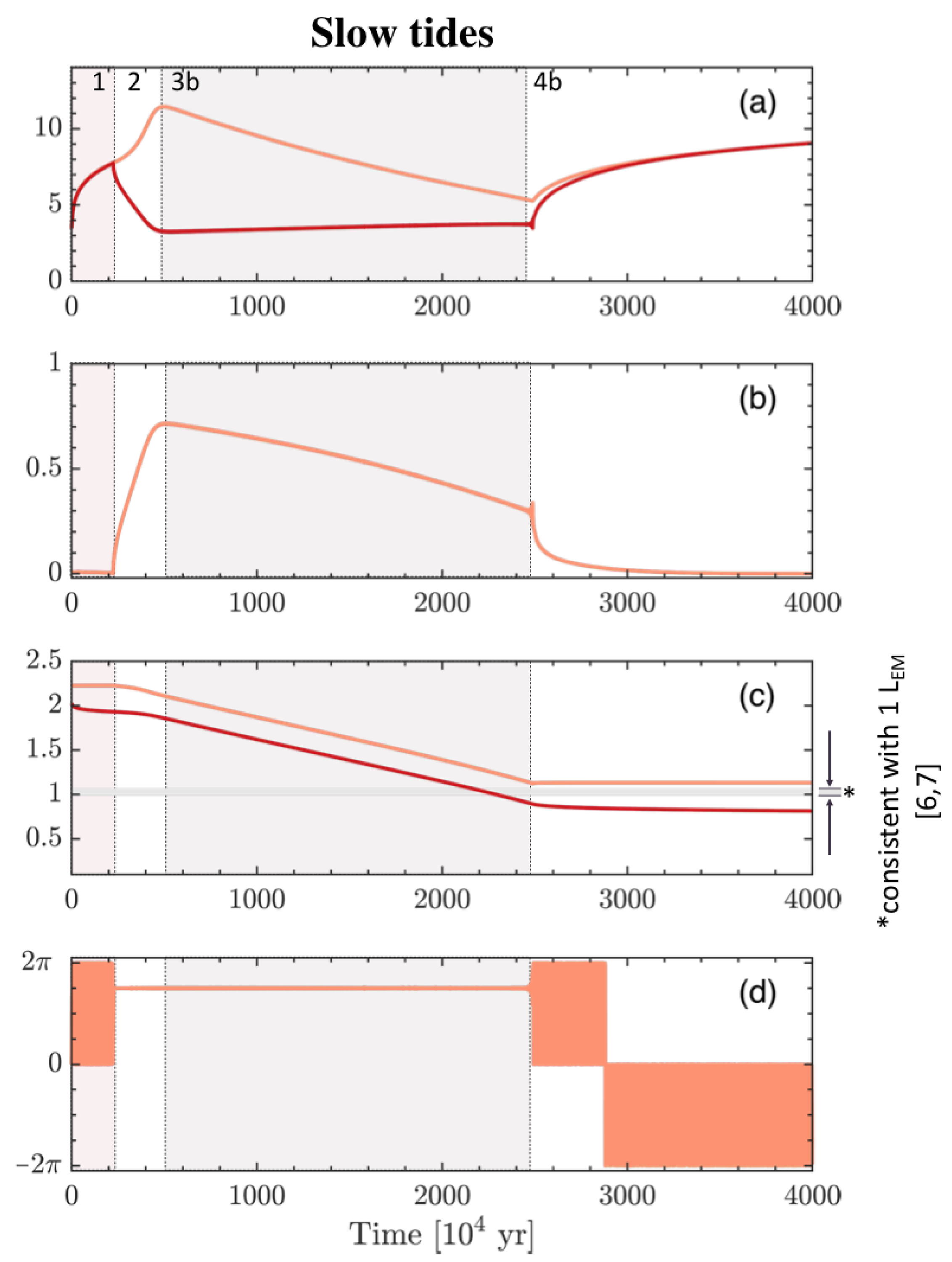




### 1 – Pre-resonance capture

Tides control the evolution, angular momentum is conserved until capture into resonance at $a \sim 7.8\,R_{\oplus}$.

### 2 – Resonance, orbit expansion

$e$ increases rapidly and the resonance angle librates around $\pi/2$. Lunar orbital expansion stalls at a critical $e$ for which expansion due to Earth's tides is balanced by contraction due to lunar tides.




### 3a – Quasi-Resonance (QR)

$\Phi$ librations increase and the system escapes the resonance to the high $e$/low $a$ side of the resonance. The Moon enters a quasi-resonance phase, in which $e$ oscillates and $\Phi$ does not librate about a fixed value. Nearly all AM loss occurs in this phase. This phase is reminiscent of the limit cycle found by Wisdom & Tian [8].

### 3b – Resonance, orbit contraction

The system remains in resonance during orbit contraction and AM removal is controlled purely by evection, the type of evolution found by Ćuk & Stewart [1].

### 4a – Exit QR phase

After exiting QR, tides dominate the dynamics and AM is conserved. There is no preference for exiting QR when the system reaches $\sim 1 L_{EM}$.

### 4b – Exit resonance

Libration grows and the system exists the resonance. We do not find a preference for resonance escape near $\sim 1 L_{EM}$, in contrast to Ćuk & Stewart [1].

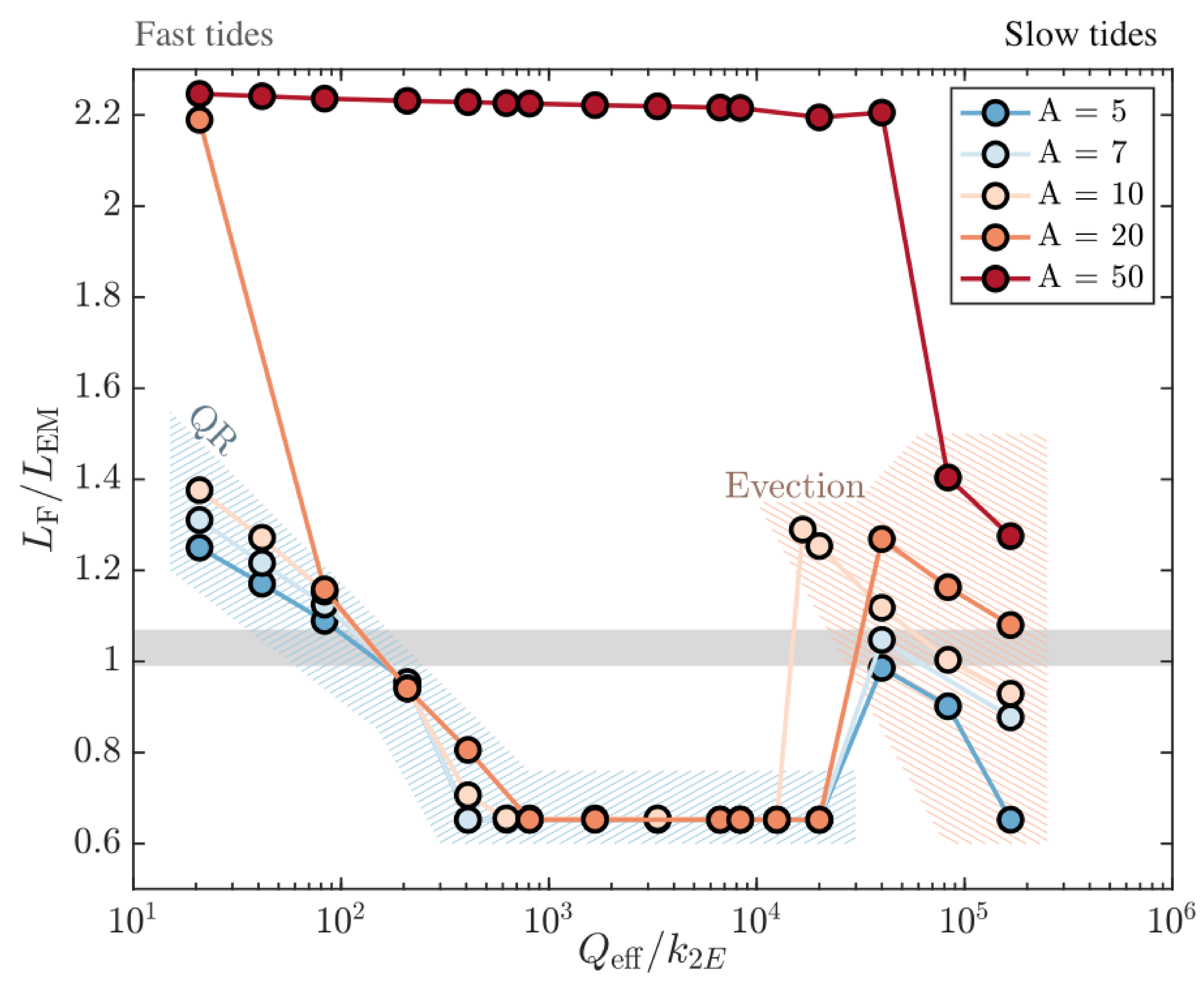
## Angular Momentum Drain

Increasing the relative tidal dissipation in the Moon and Earth, **A**, results in an earlier balance between lunar tides (which decrease $a$) and Earth tides (which increase $a$), therefore orbit contraction occurs earlier and at a lower $e$. Overall, for a given tidal dissipation value, **increasing A, would decrease the amount of AM removal**.

Our results do not show a preference for obtaining a final AM of $\sim 1\,L_{EM}$. Most cases leave too much or too little AM.

Final AM $\sim 1\,L_{EM}$ can result from either the QR or pure evection mode, each requiring a particular values for both A and $Q_{\text{eff}}/k_{2E}$.

As long as evection resonance/QR phase is occupied, the system approaches the **co-synchronous** state, independent of the initial AM.




**Final AM** - The final AM as a function of the initial terrestrial tidal dissipation factor, $Q_{\text{eff}}$ (normalized by Earth's love number, $k_{2E}$) for different relative tidal strength values, A (colors in legend). AM removal by a quasi-resonance (QR) vs. formal evection is marked by the shaded blue/red areas


**References:**

[1] Ćuk M. & Stewart S. T. (2012) Science 338, 1047-1052; [2] Canup R. M. (2012) Science 338, 1052-1055; [3] Lock S. J. et al. (2018) JGR 123(4), 910-951; [4] Touma J. & Wisdom J. (1998) AJ 115, 1653-1663; [5] Ward W. R. & Canup R. M. (2013) LPSC XLIV 3029; [6] Bottke W. F. et al. (2010) Science 330, 1527; [7] Canup R. M. (2008) Icarus 196, 518-538; [8] Wisdom J. & Tian Z. (2015) Icarus 256, 138-146.