

# Elastic Flexure Around Sputnik Planitia, Pluto, and Evidence for a Very High Heat Flux

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## Introduction

### Sputnik Planitia

- Large teardrop-shaped basin located on Pluto at 20°N 180°E
- Size: 1300 km by 900 km
- Depth: 3-4 km (basin)
- 10 km deposit of nitrogen ice
- Water ice basement

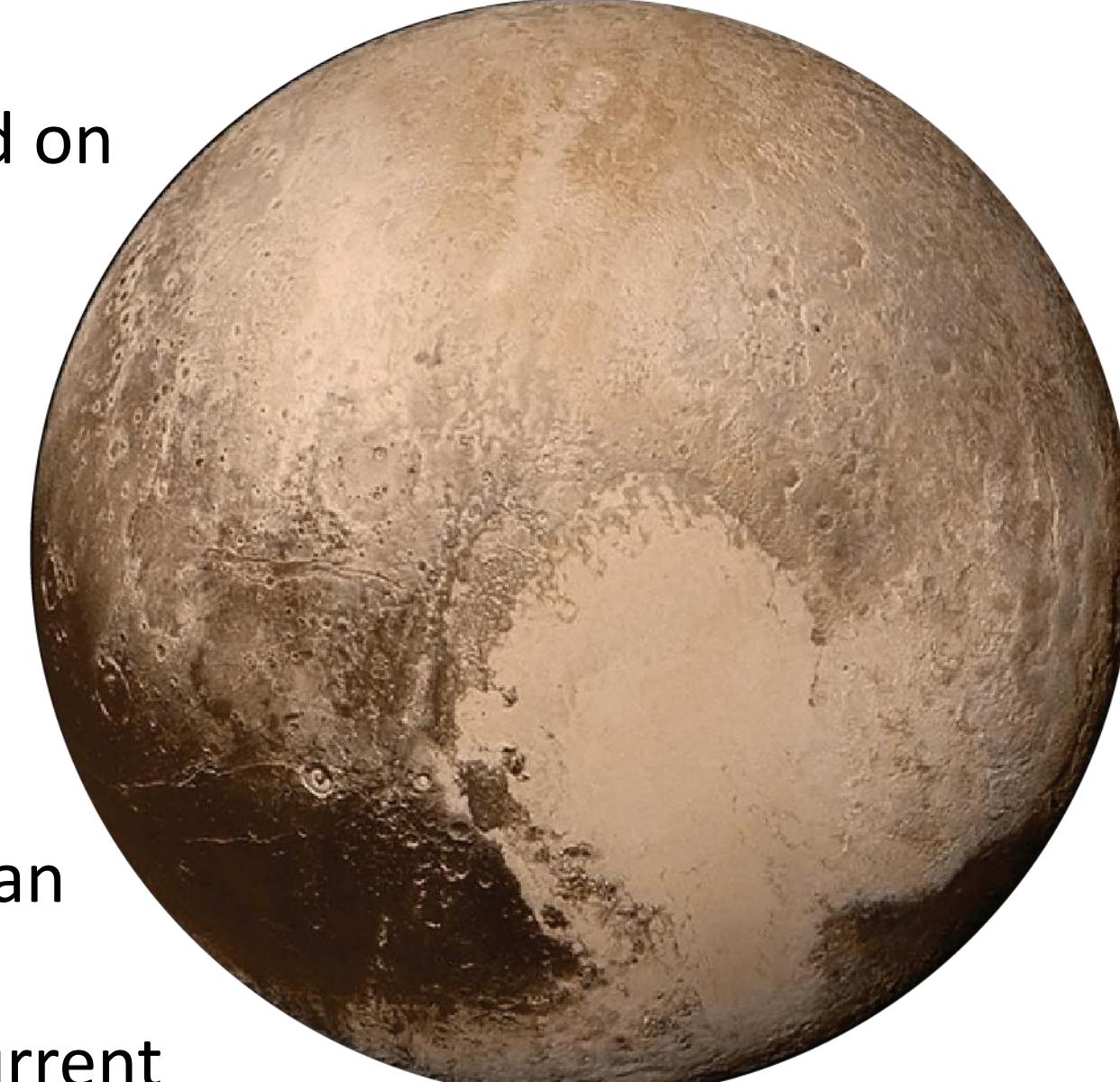


Fig.1. Global view of Pluto from the Long Range Reconnaissance Imager (LORRI) with color information from the Ralph instrument aboard NASA's New Horizons spacecraft. Sputnik Planitia appears as the light-colored region at the center (NASA/APL/SwRI).

### Formation Hypotheses

- An ancient impact basin created by an impactor later filled with N<sub>2</sub> ice. The feature would have moved to the current location through polar wander. [Keane et al., 2016; Nimmo et al., 2016].
- Runaway deposition of N<sub>2</sub> ice due to albedo feedback at the ±30° [Hamilton et al., 2016]. The depression is due to elastic flexure under the load of a thick N<sub>2</sub> ice cap. Most of the cap has since sublimated away

## Hypothesis

The topography of Pluto around Sputnik Planitia matches the prediction of an elastic plate flexed by a large nitrogen ice load

## Flexure Modeling

### Flexure Model

- The deflection  $w$  of an elastic plate in two dimensions obeys:

$$D \frac{d^4 w}{dx^4} + \rho_m g w = V_0$$

where  $\alpha = \left[ \frac{4D}{\rho_m g} \right]^{1/4}$  is the flexural parameter,  $D = \frac{E * h^3}{12(1-\nu^2)}$  is the flexural rigidity,  $\rho_m$  is the density of the layer underlying the ice shell,  $g$  is the gravity on Pluto,  $E$  is the Young's modulus of water ice,  $h$  is the elastic thickness, and  $\nu$  is Poisson's ratio.

For a single vertical load at  $x = 0$ , the boundary conditions are  $D \frac{d^3 w}{dx^3} = \frac{1}{2} V_0$  and  $\frac{dw}{dx} = 0$

- The deflection due to several line loads is found by superposition:

$$w = \sum_i \frac{V_i \alpha^3}{8D} \left( \sin \frac{x - x_i}{\alpha} + \cos \frac{x - x_i}{\alpha} \right) \exp \left( -\frac{|x - x_i|}{\alpha} \right)$$

where  $\{V_i\}$  is the magnitude of the loads at position  $\{x_i\}$

### Inversion

- The load vector  $\mathbf{V}$  that produces the topography  $\mathbf{w}$  is found by least squares optimization:

$$\mathbf{V} = (\mathbf{M}' \mathbf{M} + \mathbf{C})^{-1} (\mathbf{M}' \mathbf{w})$$

where  $\mathbf{M}$  is the operator matrix that links a load at position  $x_j$  to deflection at a point  $x_i$

$$M_{ij} = \frac{\alpha^3}{8D} \left( \sin \frac{x_i - x_j}{\alpha} + \cos \frac{x_i - x_j}{\alpha} \right) \exp \left( -\frac{|x_i - x_j|}{\alpha} \right)$$

And  $\mathbf{C} = \sigma_m^{-2} \delta_{ij}$  is the covariance matrix and  $\sigma_m$  is the mass variance

### Best Fit

- Test multiple flexural parameters to find the best fit flexure with the data through  $\chi^2$  optimization:

$$\chi^2 = \sum_j \frac{(t_j - w_j)^2}{\sigma_t^2}$$

where  $t_j$  is topography data,  $w_j$  is the predicted deflection, and  $\sigma_t$  is the noise of the topographic data

- Uncertainty of the flexural parameter is determined by the range of  $\alpha$  values for which  $\chi^2$  exceeds the minimum  $\chi^2$  by less than a threshold value  $\Delta\chi^2$  associated with the confidence limit  $p = 68\%$  (1-sigma uncertainty).  $\Delta\chi^2$  is given by the inverse incomplete gamma function, i.e., by solving:

$$1 - p = P \left( \frac{n}{2}, \frac{\Delta\chi^2}{2} \right),$$

Where  $P \equiv \int_0^x e^{-t} t^{n-1} dt / \int_0^\infty e^{-t} t^{n-1} dt$  is the incomplete gamma function and  $n$  is the number of fitted parameters.

### Elastic Thickness

- Elastic thickness is evaluated by rearrangement the formula for flexural rigidity:

$$D = \frac{\rho_m g \alpha^4}{4}$$

$$h = \frac{12D(1-\nu^2)}{E^{1/3}}$$

