Introduction: Density distribution within a small solar system body becomes important clue to understand its interior structure and origin. Interior density distribution encodes information about inhomogeneity within the small body. An offset of its center-of-mass and its center-of-figure, for example, are associated with the first degree of the gravity coefficients of the small body. Actually, a few degree and order of the gravity harmonic coefficients are important goals in a small body mission. Interior density distribution is an important to understand its interior structure of a small body and make a constraint on its formation process.

In addition, we can calculate more precise gravity field of a small body if we know the “real” density distribution within it. A method to estimate interior density distribution is also helpful for other dynamical studies on perturbation of ejecta trajectories released from the surface of the small body or a spacecraft trajectory close to the small body.

The best way to constrain interior density distribution is to obtain higher degree and order of the gravity harmonic coefficients from orbital determination of a spacecraft. The Near Earth Asteroid Rendezvouz (NEAR) mission precisely obtained the coefficients of asteroid 433 Eros at least up to several degree and order [1]. On the other hand, only the order 0 coefficient was estimated in the Hayabusa mission to asteroid 25143 Itokawa because of some difficulties in the orbital determination process [2].

In a real mission to a small body, we do not always observe the exterior gravity field up to enough high degree and order. In this study, we propose a method to make a constrain on interior density distribution, based on the equilibrium state between topography and the gravity field on the surface of the asteroid.

Self-Correcting Assumption: Regolith migration in the down-slope direction is a dominant process to refresh the asteroid surface. Loose regolith or small gravel tend to move to areas of low potential. If there are plenty of mobile regolith layer and down-slope disturbance source such as seismic shaking due to meteorite impacts, the surface topography of an asteroid becomes close to a kind of the equi-potential surface over sufficient time. For example, smooth terrain which is covered with centimeter sized gravel is located in polar regions of asteroid Itokawa. It is seemed to be a result of this erosion process on the asteroid [3].

Mean density estimation. A method to estimate asteroid’s mean density was investigated on the assumption of a self-correcting system as mentioned above [4]. The gravity field on the asteroid surface can be expressed as a function of total mass (mean density) of the asteroid. If an object spinning at a rate of several hours is covered with plenty of fluid, its equilibrium shape would be a spheroid with a smooth surface. If the self-correcting system is working well also on an asteroid and surface topography is enough relaxed, the variance of the gravity potential represent a minimum when the “real” mean density is assumed. We can recognize

Density distribution estimation. In this study, we fixed asteroid’s total mass to a result of orbital determination process. Assigning different types of density distribution within the asteroid, we searched for density distribution which minimized the variance of the gravity potential. Based on the self-correcting assumption, we recognized the minimum as an estimation result.

Gravity Calculation: We used a polyhedral shape model for the gravity calculation. In order to represent density variations, we generated numerical meshes also inside the polyhedron. As possible density variations, we tested a planar division model (the compressed-head model for Itokawa), where the polyhedron has different densities on each side of a planar boundary, and single/double core models, where the asteroid has a core(s) of high density (low porosity) within it.

After assigning density distribution to each part of the polyhedron, we accumulated contributions of all volume meshes to obtain the surface gravity field. Then, we calculated the variance of the gravity potential through the polyhedron’s surface. For example, the potential variance in the compressed-head model is expressed as a function of density of Itokawa’s “head” part.

$$\sigma^2(\rho_{\text{head}}) = \frac{1}{N_s - 1} \sum A_j \left( \frac{U_j(\rho_{\text{head}})}{U_{\text{ave}}(\rho_{\text{head}})} - 1 \right)^2$$

where $N_s$ is the number of the polyhedron’s surface facets, $A_j$ and $U_j$ is an area and the gravity potential of $j$th surface facet and $U_{\text{ave}}$ is the potential average through the polyhedron’s surface.

Application to Eros and Itokawa: Asteroid Eros is a good sample for this kind of study because we know its interior homogeneity from the exterior gravity field to some extent. At first, we tested the potential variance minimization technique using an Eros’ shape
model (34.4 × 11.2 × 11.2 km in size), and then we applied the same way to asteroid Itokawa (535 × 294 × 209 m), whose interior is unknown.

**Eros.** Figure 1 shows the result of density distribution estimation when different densities were assigned to the regions at X > 0 and X < 0 in the body-fixed frame. The result clearly shows that the standard deviation of the gravity potential (i.e. a square root of the potential variance) is minimized at the same value with the mean density of Eros. Our technique is consistent with the Eros’ homogeneous interior [1, 5].

**Itokawa.** The similar profile was obtained from the planar division (compressed head) model for Itokawa in Figure 2. However, the minimum of the potential standard deviation is at 2,730 kg/m³ of the “head” density, which is much higher than Itokawa’s mean density, 1,950 kg/m³. At this time, the “body” density corresponds to 1,870 kg/m³.

In this density distribution, the offset of its center-of-mass is about 16 m toward the “head” region. Our result is consistent with Itokawa’s spin-up due to the solar radiation. A great center-of-mass offset by 21 m was estimated by a ground-based light curve observation and thermophysical simulation using Itokawa’s shape [6].

If our technique works properly also for a subkilometer-sized asteroid, it is possible that Itokawa has density variations within it. The above result corresponds to a wide range of porosity from 15% in the “head” region to 40% in the “body”. One of possible scenarios to explain such a great variation is that a large fragment was taken into the Itokawa’s “head” during a reconfiguration process after its parent body was broken up into pieces by a catastrophic collision.

The potential variance minimization technique can be applied to small bodies whose exterior gravity fields are not measured. It is possible to be helpful for small body missions in the future.