Background: Giant Impacts (GIs) are prevalent in the last stages of planet formation, where planetary ‘embryos’ with masses near that of Mars (~0.1 M\textsubscript{Earth}) undergo gravitational instability. This late-stage, unstable configuration produces several close dynamical encounters that lead to GIs. GIs are modeled in a Smoothed Particle Hydrodynamics (SPH) code, which treats the planets as Lagrangian elements with thermodynamics described by an equation of state and, if strength is important, with a rheological model.

Since GIs operate at gravitational timescales \( t_{\text{grav}} \sim (\text{Gyr})^{3/2} \), collisions occur on the order of hours, and thus ample SPH modeling time is required in order to resolve the collisions, as well as the debris dynamics which may include long-period, quasi-stable orbits. Moreover, the setup process of SPH simulations is non-trivial and can involve several steps. Initializing different thermal or morphological regimes such as molten planets and/or multi-layered planets, which are expected in the planet formation process, require special treatment in the setup procedures; this is especially true for tabulated equations of states. Often SPH simulations of \( \sim 10^5 \text{–} 10^6 \) particles require several days to complete, which makes incorporating them directly into \( N \)-body planet formation simulations taxing, since for every GI, the \( N \)-body simulation must wait for the SPH simulation to be setup and completed.

From analysis of simulation outcomes [1] we have generated scaling laws in the form of algebraic expressions that predict the outcomes of these collisions for a large variety of pre-impact conditions expected in the embryo phase of planet formation. These scaling laws are similar to that of [2,3], with a few key exceptions:

1. the simulation database includes high-parameter resolution in the velocity range between 1-2 \( v_{\text{esc}} \) (in increments as low as \( 0.05 \) \( v_{\text{esc}} \)), the most common impact velocities for a self-stirred population of planetesimals,

2. the parameter space samples water-mantled planets, similar to [4], but includes a larger range of impact angles.

The advantages of 1. and 2. allow us to pay particular attention to the transition between merging and hit-and-run for GIs. In addition, through the development of these scaling laws, we placed emphasis on the simplicity and ease of adaptation into \( N \)-body codes.

Methods: We utilize results from [1], a large set of SPH GI simulations that use the ANEOS equations of state for Quartz, Forsterite, and Iron. The database covers a range of impact velocities from 1-5 times the mutual escape velocity of the impacting planets. The materials covered include homogeneous silicate planets, 2-layer silicate-iron planets, and 3-layer water-silicate-iron planets. The silicate-iron mass ratio is 0.7, characteristic of terrestrial ‘chondritic’ abundances. These simulations were analyzed in terms of pre- and post-impact conditions, including final mass of the largest remnants, escaped mass, change in internal energy, and angular momentum.

Results: We find that the bulk outcome of giant impacts, especially in terms of the mass of the largest remnant, are not sensitive to the scale of the collisions [e.g., 2,5]. This result is consistent with previous findings [2] which modeled GIs as assemblages of hard-spheres with different orders of magnitude in total mass. Moreover, we examined the grazing criteria of [5] which describes the impact angle at which hit-and-run collisions would occur for a given set of impacting bodies. This criteria is necessary in order to adopt hit-and-run outcomes into \( N \)-body planet formation codes.

![Figure 1: Shown is our phase space for giant impacts. Outcomes are divided into ‘Merger’ or ‘Hit-and-run’; both involve various degrees of erosion. The red line is the grazing criteria of [5 Eq. 9] and the black vertical line is the grazing relation of this study. The black dotted curve is the velocity criterion from [8 Eq. 16] and the black solid curve is the velocity criterion developed in this study.](image)
[e.g. 6,8]. We find the relation of [5] is sufficient for impactor-to-target mass ratios < 0.2, but the relation underestimates hit-and-run collisions at larger impactor-to-target mass ratios and overestimates them at low velocities.

We also examined the criterion of [8] for predicting the hit-and-run regime and found it to be robust across various material types and impact scales. However, we caution the use of this relation alone for determining the prevalence of hit-and-run collisions as it allows for hit-and-run collisions below the grazing angle (see Figure 1). At these low angles, the second largest remnant is a mix of thoroughly disrupted material from the impactor and debris from the target. In contrast, we define a hit-and-run collision as one that leaves the impactor mostly intact. Since the thermodynamic consequences for a remnant that was disrupted and gravitationally reaccumulated downrange is significant as compared to one that is remains largely intact through the collision, we find such a prescription for hit-and-run collisions necessary.

We expect the adoption of our updated predictions for the hit-and-run transition regime to affect N-body simulations differently at various epochs or scenarios in terrestrial planet formation. Early, where embryo-embryo gravitational scattering leads to impact velocities \( \sim 1-1.5 \ v_{\text{esc}} \), hit-and-run outcomes will be less prevalent as compared to [6,7] which implemented the model of [5]. At late epochs, where impact velocities increase due to dynamical stirring by few, large planets, e.g., the late ‘oligarchic growth’ phase, the model employed in [6,7] is likely to underpredict the prevalence of hit-and-run GIs.

Our scaling laws in general reflect the findings of [9], which suggest that accurately accounting for the occurrence of hit-and-run collisions requires a prescription for the grazing angle as well as the critical velocity at which the projectile escapes downrange, as opposed to grazing and merging. We formulated the scaling laws across several material types and density stratifications. Furthermore, these scaling laws were developed on a dataset with high resolution of both impact angle and velocity, especially in the regimes most probable in late-stage planet formation. Ongoing work includes implementing these scaling laws to track the material abundances of growing terrestrial planets.

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