STREAMING INSTABILITY IN TURBULENT PROTOPLANETARY DISKS: THEORETICAL PREDICTIONS.

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Introduction: Planetesimal growth from dust particles in protoplanetary disks (pp-disk) occurs during two broad phases. During the early epoch (t_{disk}<1Ma), where the mass of the gas disk dominates the dusty component, the gas phase most likely experiences some kind of sustained turbulent activity [1,2] which drives the kinematics and growth of small-sized dust particles (~1-100 micron). Beyond this early stage theoretical understanding of the growth of particles beyond this size is uncertain: particle growth is throttled by several "barriers" to growth including the bouncing, fragmentation and radial drift barriers, among others, e.g. see discussion in [3]. It is one of the longstanding challenges in planet formation theory to explain how, under these conditions, 10-100 km sized planetesimals are constructed in under 0.1–0.5 Ma [3-6].

Several approaches have been proposed to solve this puzzle in which the Streaming Instability,"SI" hereafter, has received much attention [5,7]. The SI operates by concentrating particles via a two-way process involving momentum exchange via drag between the particles, treated as a collective pressure-less fluid, and an ambient Keplerian sheared gas. SI effectively concentrates particles when (i) the background flow is nearly laminar, (ii) for Stokes numbers τ_s near 1, and (iii) when the ratio of the local volume mass density of particles to gas (i.e., $\epsilon \equiv \rho_p/\rho_g$) is also near 1.

Note that drag relaxation timescales (scaled by the local orbit times) are measured by τ_s, which, in turn, is directly proportional to particle size – thus for the SI to be most effective, particles must be large enough and plentiful enough (locally speaking), and drag forcing and Coriolis effects must operate on the same timescales but in differing directions.

The viability of the SI in producing enhanced particle concentrations during the early epoch of pp-disk evolution is not certain for several reasons: (A) pp-disk fluid simulations modeling various turbulence driving instabilities, e.g.,[8-10], indicate all such processes generate turbulent activity in Dead Zones in the range of $\alpha_t \sim 10^{-5}$-10^{-3}, where α_t measures the intensity of the underyling turbulence, (B) particle growth evolution modeling shows that during the early epoch, particles rarely achieve values of τ_s much greater than 0.05 (at best). In other words, one expects that a turbulent medium should act against particle concentration and a nascent stage of particle growth means strong coupling between components which implies that drag and Coriolis effects are operating on differing timescales during the early epoch. *All of these considerations suggest that the SI may be greatly diminished or extinquished under modest amounts of turbulence at too early a stage in pp-disk evolution*. To date, there are a handful of dedicated numerical simulations devoted to assessing how SI operates in a turbulent medium, e.g., [11-13]. The implications if these studies are inconclusive.

Aims, Methods and Inputs. *Aims*. Numerical simulations of the SI in turbulent media is an expensive endeavor. Following simple arguments it will be of utility to have a theoretical framework that predicts the character of and degree to which the SI operates in a simplified theoretical model that represents the effect of turbulence within a local section of a pp-disk.

Methods. A simple model of turbulence is added to the framework adopted in the original SI analysis done in [7], which presents an examination of a local (point) linear stability analysis of an inviscid 2-fluid system in a laminar shearing box setting under axisymmetric conditions. We expand this analysis by adding to it a model of turbulence that is assumed isotropic and appears in the 2-fluid model both (i) as a turbulent viscosity influencing the gas momentum via the formulation $\alpha_t c_s H \nabla^2 \mathbf{u}_g$ and, (ii) as a turbulent diffusion acting upon the dust concentration and is given as a source term in the dust mass continuity equation as $\alpha_t c_s H (1+\tau_s^2)^{-1} \nabla^2 \rho_p$. The local disk scale height is H.

Inputs. The point linear stability analysis assumes axisymmetric perturbations, i.e., exp $i(k_x + k_z - \omega t)$ and solutions to the temporal response ω are determined as a function of α_t, τ_s, and ϵ . $\lambda_x = 2\pi/k_x$, $\lambda_z = 2\pi/k_z$ are, respectively, the disturbance wavelengths in the radial and vertical directions. Other parameters is the disk opening angle, $\delta = c_s/V_k$, where c_s is the local soundspeed and V_k is the local Keplerian speed. To represent the effect of turbulent vertical dispersal of a pp-disk, we adopt a constrained functional form for the local dust-to gas mass ratio via the form $\epsilon = f\sqrt{(\alpha_t + \tau_s)/\alpha_t}$, where f is the nominal initial dust to gas mass ratio (typically = 0.01). This expression represents the effect of vertical settling [3].

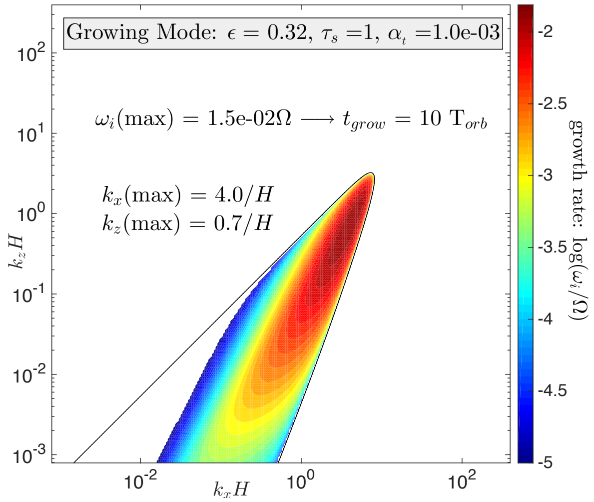

Figure 1. Growth rates of turbulent SI under turbulent conditions as reported in [11] with δ =0.04. Maximally growing wavenumbers are identified. Black line indicates boundaries of absolute growth, shaded regions show growth on timescales of 40,000 T_{orb} or less. Growth rate is Im(ω_i).

Preliminary Results: In Figure 1 we display the growth rates for disturbunces in a model meant to represent the simulations reported in [11] wherein the fate of the SI is examined in a model of MRI turbulence with α_t=10^{-3} and, τ_s=1, together with a disk opening angle δ =0.04. Figure 1b-c of [11] shows that the radial wavelength of the fastest growing is ~1.2*H,* with a growth timescale of about 10-20 T_{orb}. The SI analysis displayed in Figure 1 predicts that under these same conditions investigated in [11] the fastest growing mode should have a growth timescale of 10-20 T_{orb} with λ_x ~1.55*H*. Under these conditions the vertical scales of the modes are much larger, λ_z ~9*H,* indicating SI induced concentrations that are vertically oriented and sheet-like. These trends indicate good correpondence between the theory developed here and published numerical results of the SI under turbulent conditions.

In Figure 2 we display the results of a comprehensive scan where we identify the fastest growing mode for each given parameter pair α_t, τ_s, under the assumption that ϵ is constrained as per our prescription above (with the assumption of *f*=0.01). Indicated on the graphs are the results of [12] (red diamonds) and [11] (black triangles). We note that timescales for growth of small particles (τ_s<0.01) is rather large, on the order of many thousands of orbit times. These timescales are on the order of drainage timescales due to the radial drift barrier [3]. Two broad regimes are identified: one designating a turbulent regime, in which the growth rates are relatively low with emerging structures that are vertically oriented sheets, and second laminar regime, in which the growth rates are relatively high and the fastest growing modes are small and equidimensional in the radial and vertical directions. The dividing line appears to be determined by those values where ϵ =1, indicating a special value in which there is neither growth or decay.

A comprehensive summary and interpretation of all of these results will be presented at the time of the meeting, see also [14]. However, we see already that under modest levels of disk turbulence α_t ~ 10^{-5}-10^{-3}, it is difficult to effectively activate the SI on reasonable disk timescales for small particles, i.e., for τ_s<0.05, which is essentially the stopping time of chondrules and their aggregates [5].

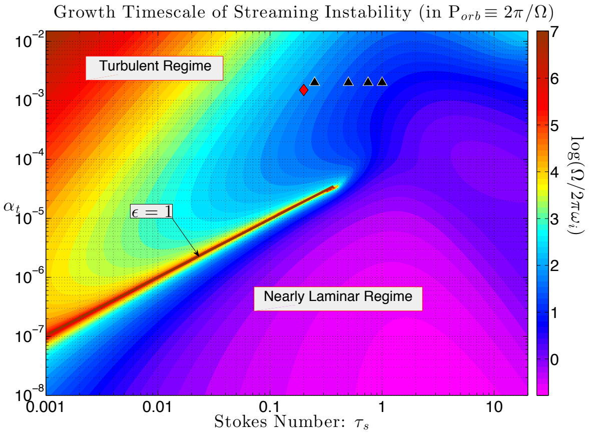

Figure 2. A comprehensive parameter sweep identifying the fastest growing mode for conditions in which δ =0.04. The turbulent regime indicates a regime in which the resulting structures are strongly vertically oriented while the nearly laminar regime corresponds to noodle like structures together with fast growth rates. Red diamond indicates results of [12] while the black triangles indicates results of [11].

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