THE INFLUENCE OF BOUNDING RADII IN A CURVED GEOMETRY, BI-MODALLY HEATED CONVECTING LAYER.

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Introduction: Convection in the cryo-shells of icy satellites and silicate mantles of rocky moons and planets is characterized by a strongly temperature-dependent viscosity that can result in the formation of an immobile (or stagnant) conductive lid at the top of the upper thermal boundary layer of the system. The onset of stagnant-lid convection is a function of system curvature which affects the mean temperature of the convecting fluid [2,5].

For stagnant-lid convection in purely basally heated systems the interior temperature of the vigorously convecting fluid reaches a temporally averaged value which is less than the temperature of the isothermal core. The heat flux at the base of the system always remains positive, and heat is always drawn from the core. For systems that are bi-modally heated, heat input from internal heating is unaffected by temperature and internal temperature increases with increasing internal heating rate even as heat flow from the core is reduced. Consequently, there exists an internal heating rate above which the temperature of the interior exceeds the temperature of the isothermal core; resulting in a negative heat flux at the base of the system [e.g., 6].

Previous convection studies [e.g., 2, 7, 8, 9, 10] considering variable curvatures have quantified how interior temperature and basal heat flux depend on geometry as well as internal heating rate. Here we consider the effect of internal heating rate on convective regime in variable core-size geometries. Additionally, we investigate the parameter space of core-size and internal heating rate to identify when heat flow from the core ceases.

Methods: Thermally driven convection in a Boussinesq fluid with infinite Prandtl number is modeled in two-dimensional spherical annulus and three-dimensional spherical shell geometries. A hybrid finite-difference/finite-volume code, StagYY, solves the governing equations nondimensionally, using a parallelized multigrid method [1, 2, 4]. Our calculations emulate bimodally heated convection in a planetary mantle or ice-shell. The surface and core-mantle boundary are modeled as free-slip in each calculation and are isothermally fixed to nondimensional temperatures of $T = 0$ and $T = 1.0$, respectively.

The controlling parameters that determine the convective regime are the ratio of core radius to outer shell radius, $f = R_c/R_p$, the reference Rayleigh number, Ra, and an internal heating rate, H. A Frank-Kamenetskii rheology law is implemented, with a reference viscosity of $\eta^* = 1.0$ at a temperature of $T = 0.5$.

Convective regimes are classified using the mobility, M, which is the ratio of surface rms-velocity to interior rms-velocity. If $M \geq 0.4$ we describe the surface of the system as mobile. If $M < 0.01$ we consider the system to be in the stagnant-lid regime [3]. Intermediate values of M correspond to a transitional regime.

Results: Our initial results use a Rayleigh number-$Ra = 3.2 \times 10^5$ and fix the viscosity contrast to $\Delta\eta_T = 1.0 \times 10^5$. We analyze stagnant-lid convection (SLC) in 2D and 3D systems with curvatures including relatively small-core shells (f as small as 0.3). Figure 1 shows the effect that curvature has on the transition to stagnant-lid regime when H is varied. Several peculiarities of convection in a strongly temperature-dependent viscosity fluid are identified for low curvature systems.

Figure 2 shows that for $f \leq 0.7$ the transition to SLC occurs for an internal heating rate in a range of H = 13-14. For $f > 0.7$ a discontinuity in the transition to SLC is observed for H, between thin shell and plane-layer geometries. This difference may be explained by the inherent difference in topology for both systems. In contrast, the transition to the regime where heat flows into the core follows a monotonic trend for increasing H with decreasing f.

Figure 3 shows selected temperature fields for various spherical annulus cases with different internal heating rate. The internal heating rates we specify for the cases shown in the figure adjust the temperature profiles to agree for different geometries. For the Ra we specify in this figure, large core systems with moderate H exhibit downwelling sheets that are absent in the Cartesian geometry case. However, small core spherical systems with relatively large H can resemble the temperature field of the plane-layer. In summary, although the temperature profiles are similar, the flow patterns and hence convective regime may differ when curvature is present.

In our previous investigation with purely basal heating we found major differences between 2D and 3D geometries when f was small. Hence, we examine the agreement of our bi-modally heated results in spherical annulus geometry with the fully 3D spherical shell geometry. We extend our initial results to map the transition to stagnant-lid convection and the cessation of heat flow into the mantle from the core when Rayleigh number $Ra = 1.0 \times 10^4$.

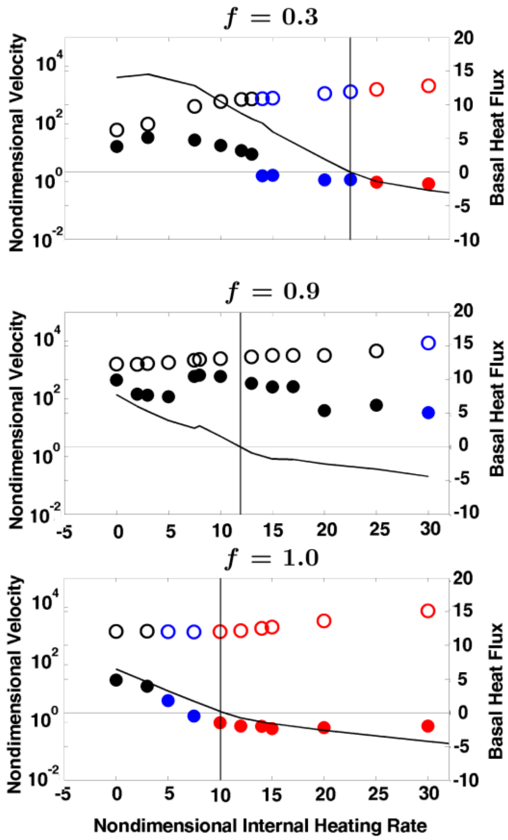

Figure 1. *Nondimensional basal heat flux (solid line right axes), volume-averaged rms-velocity (open circles left axes) and surface rms-velocity (closed circles left axes), as a function of nondimensional internal heating rate for systems with a fixed viscosity contrast $\Delta\eta_T = 10^5$ and $f = 0.3$ (top), $f = 0.9$ (middle), and $f = 1.0$ (bottom). Black circles indicate $0.01 < M < 0.4$, blue circles indicate $M < 0.01$ and red circles denote $M < 0.01$ and $V_{surf} < 1$. Calculations employ basal and internal heating with $0 \leq H \leq 30$. Vertical lines indicate the value of H when heat flow from the core to the mantle ceases*

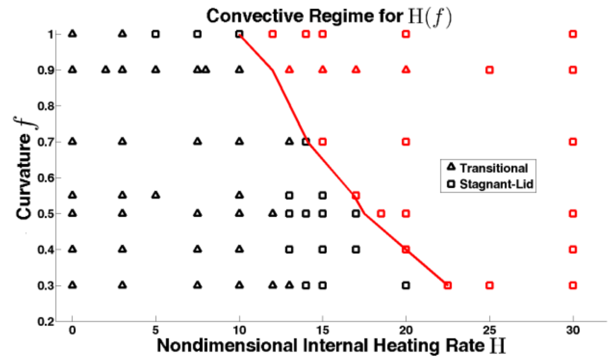

Figure 2. *Covective regimes for bimodally heated calculations are plot as a function of curvature f and nondimensional internal heating rate H. Red indicates cases where heat flows into the isothermal core. The red curve values are determined from the approximate H values for which heat flow from the core to the mantle ceases. (see vertical lines in Figure 1).*

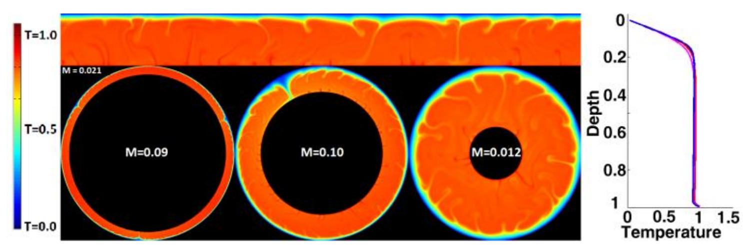

Figure 3. *Selected temperature field snapshots and geotherms for Cartesian and spherical geometries with bi-modal heating. Viscosity contrast is $\Delta\eta_T = 10^5$. Mobility values are inset in white. Temperature profile colors are: black (f=1.0, H = 0); red (f=0.9, H = 2); magenta (f=0.7, H = 3) and blue (f=0.3, H = 13). The grid resolutions for each case are: (128 × 1280) (f=1.0), (128 × 7680) (f=0.9), (128 × 2560) (f=0.7), and (128 × 1024) (f=0.3).*

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